CIRP – SMS Steering Model Workshop

ADCIRC – Grays Harbor Grid Design and Parameter Selection

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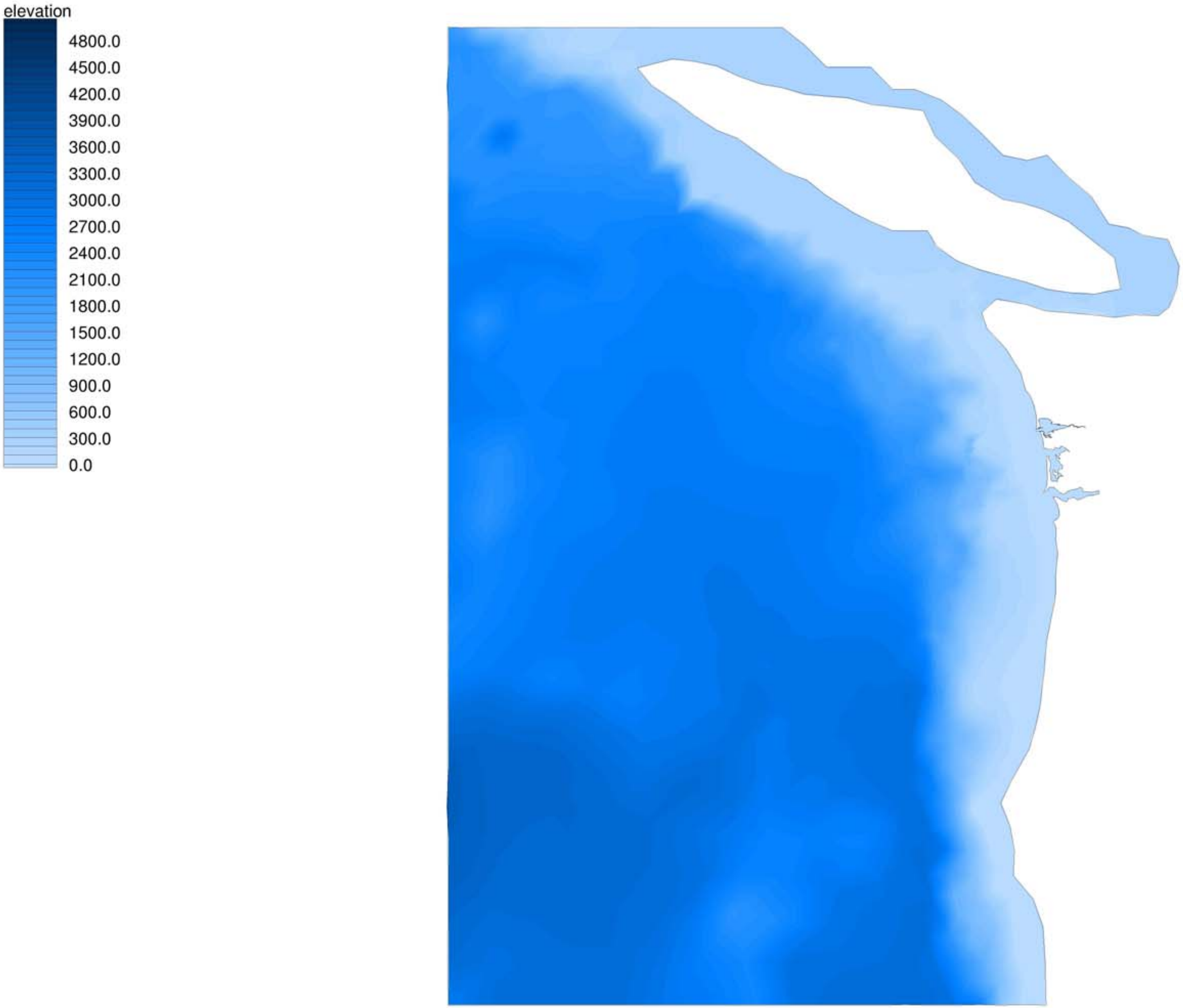
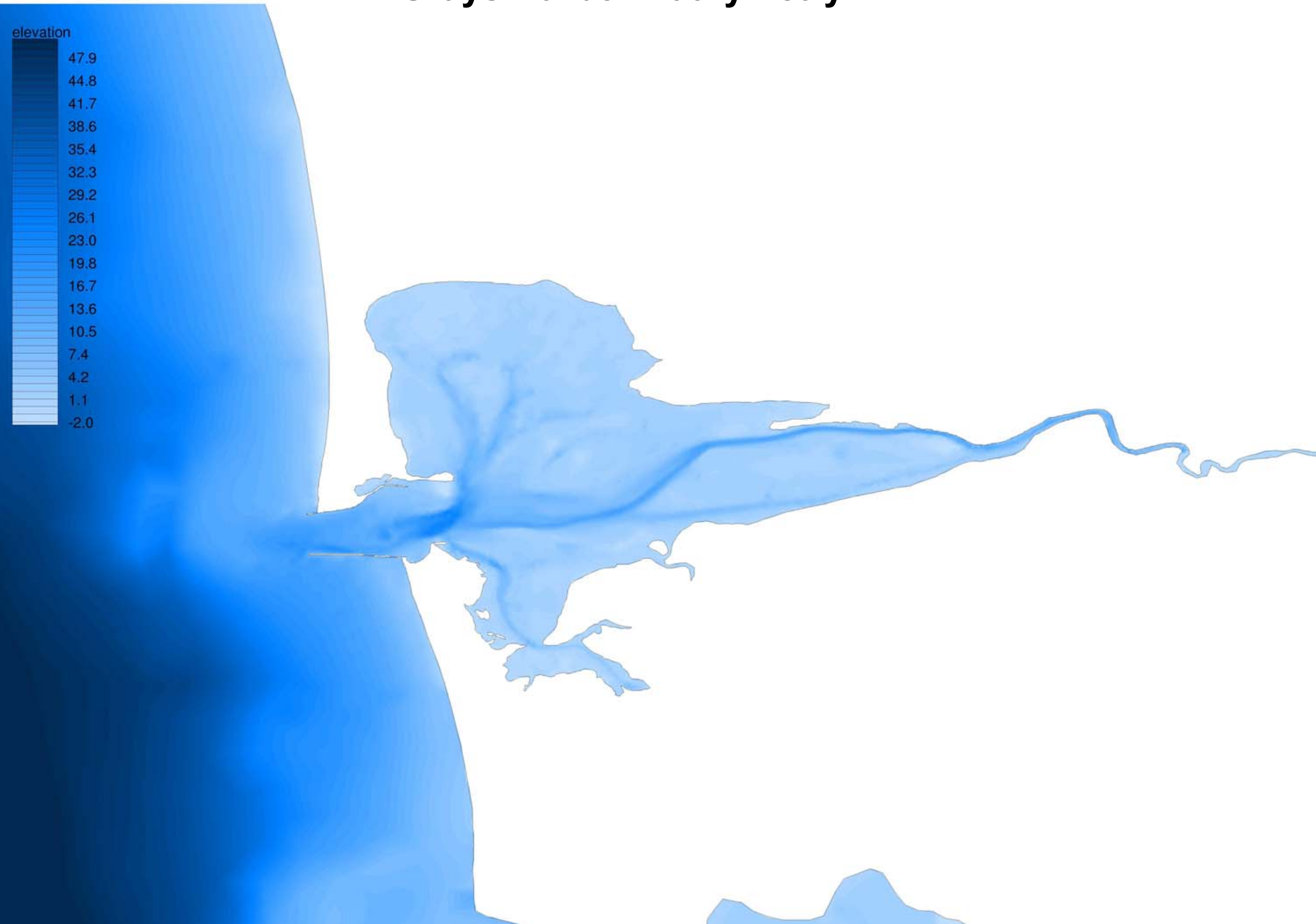
Grays Harbor Application

- Grays Harbor is an inlet connected to the Pacific Ocean in the State of Washington
- Example application features
  - Tidally driven
  - Partially submerged jetty dominates flow structure
  - Flow through deep channel
  - Large and very distinct eddies form through the tidal cycle
- Important implementation considerations
  - Sufficient grid resolution must be provided
  - Time step must be set correctly
  - It is critical to correctly set the GWCE τ_0 parameter.

Grays Harbor Bathymetry



Grays Harbor Bathymetry



Grays Harbor Bathymetry

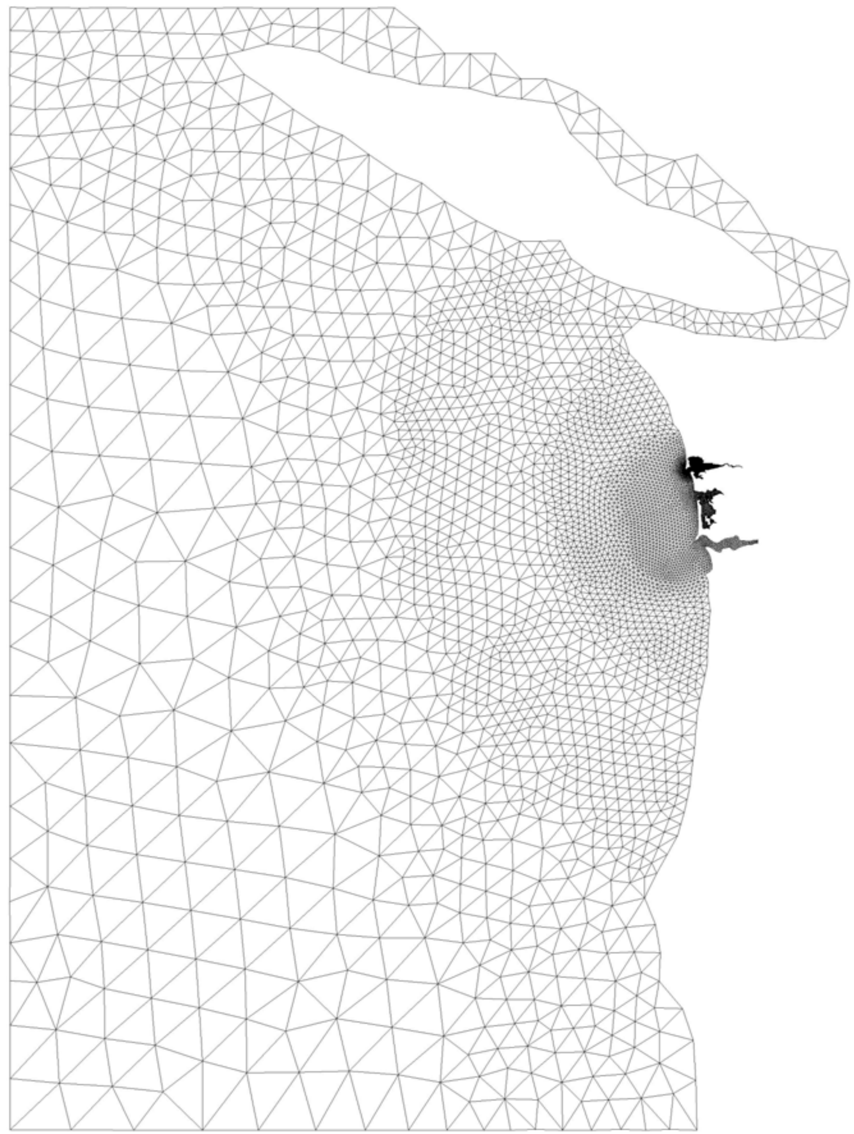
Spatial Grid Design

- Important grid design considerations
  - All processes that contain significant energy must be resolved in order for the computation to be correct
  - In under-resolved grids, the physics of the flow may try to feed energy into the unresolved scales
  - Note that the smallest resolvable scale is the $2 \cdot \Delta x$ length scale although in practice better resolution is required
  - Wiggles can be a manifestation of insufficient spatial resolution (note that they can also be associated with instabilities as well as folded dispersion behavior)
  - “Don’t suppress the wiggles, they’re telling you something,” Gresho, 1979

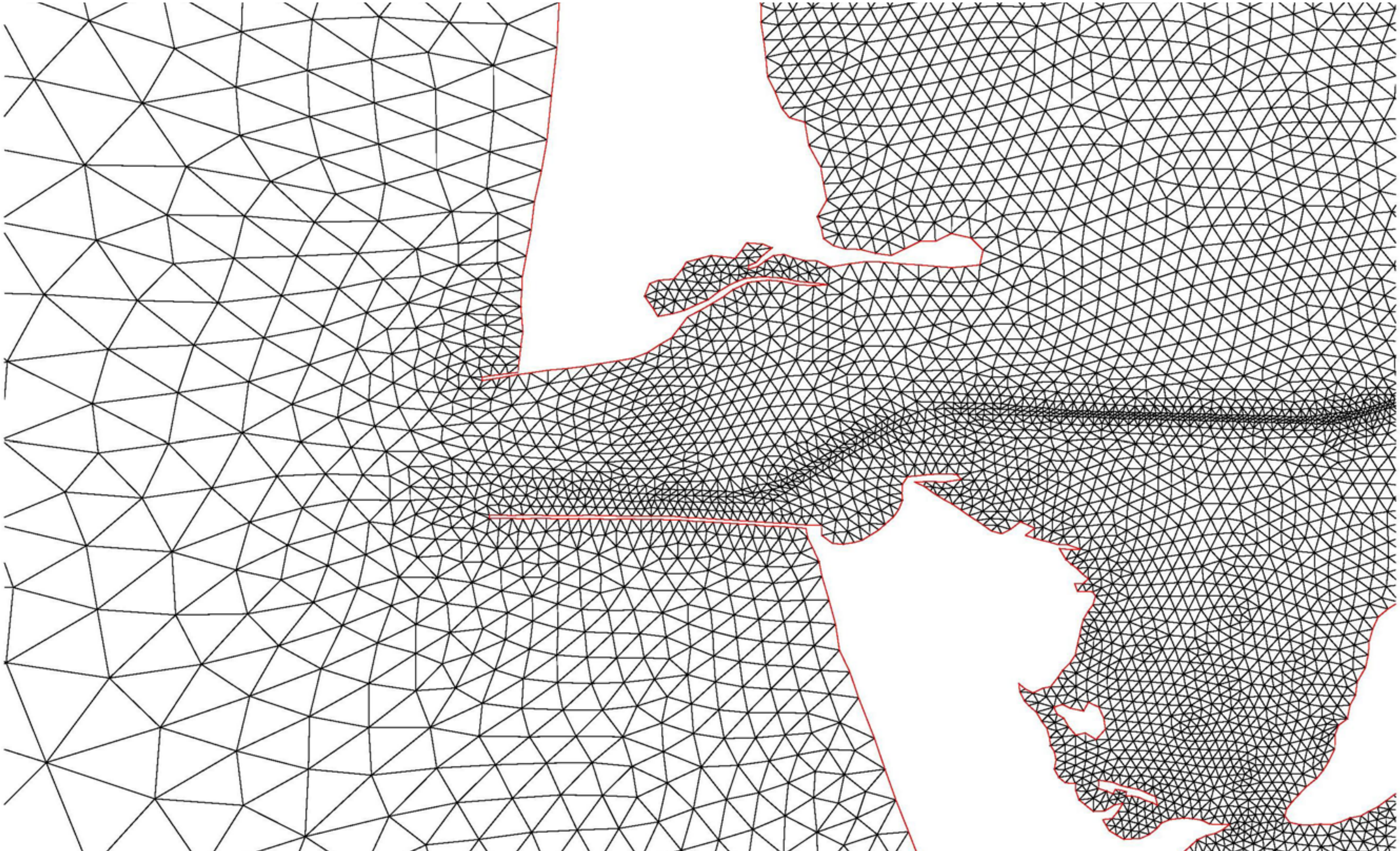
Spatial Grid Design

- Grays Harbor grid implementation considerations
  - Near the coast in complex regions such as inlets, the wavelength to grid size, $\lambda / \Delta x$, criterion is entirely insufficient for determining the length scale to grid size ratio of the flow
  - Length scales are defined by the geometry, bathymetry and the details of the hydrodynamics
  - Sufficient grid resolution is critical to capturing the formation of eddies and cross barrier flow patterns

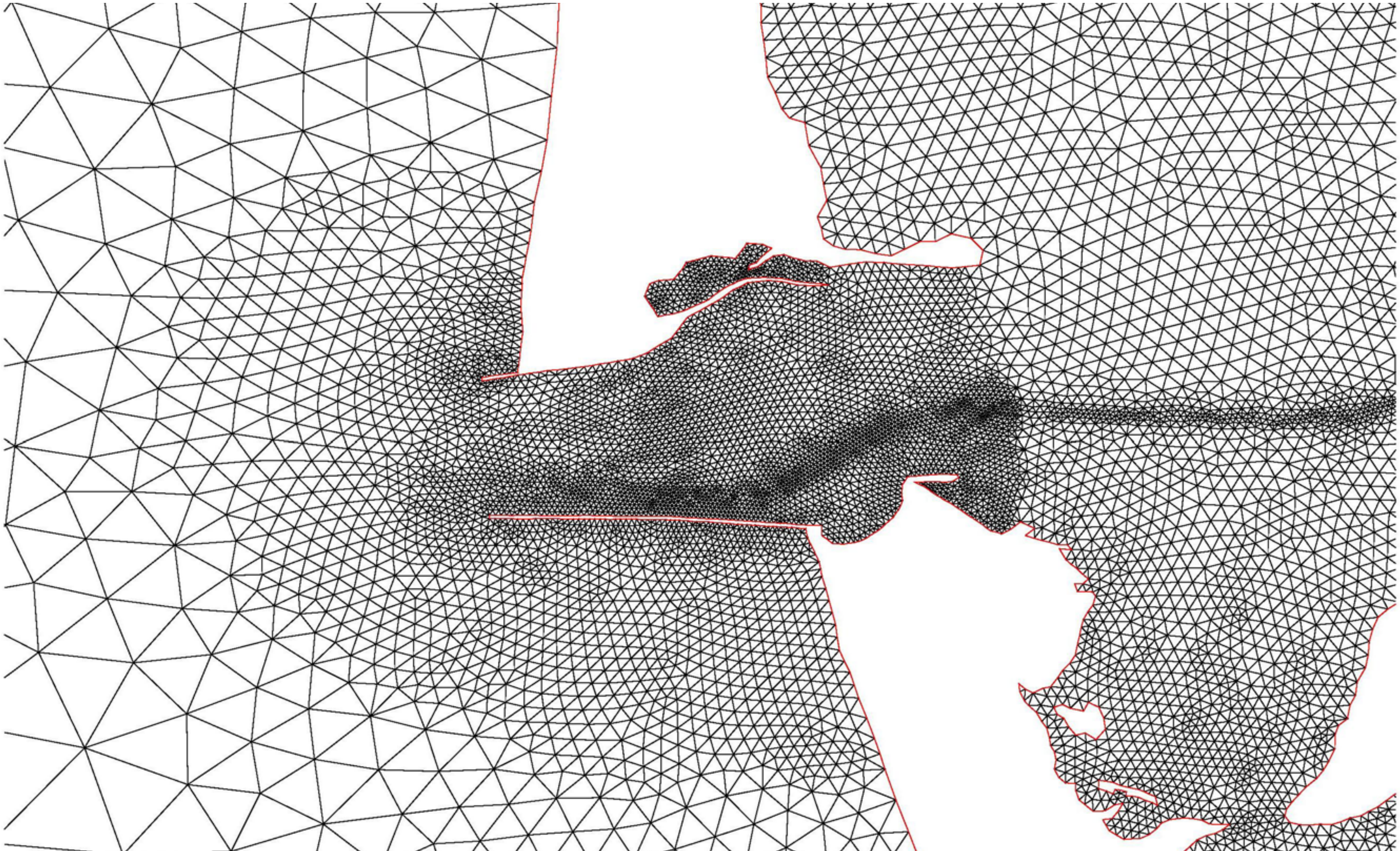
Grays Harbor Coarse Grid



Grays Harbor Coarse Grid in Inlet Vicinity



Grays Harbor Fine Grid in Inlet Vicinity



Spatial Grid Design

- Comparison of coarse grid and fine grid in the vicinity of the inlet
  - Movie of coarse grid flow
  - Movie of fine grid flow

Spatial Grid Design

- It is critical to provide sufficient resolution in inlet regions
  - Geometry and channel bathymetry must be well resolved in the grid
  - Hydrodynamic features such as important eddies must be resolved with at least 5 to 7 nodes across an eddy.
  - Node to node spatial oscillations can easily appear as a manifestation of a poorly resolved flow
    - The adequacy of the spatial resolution must then be examined
    - Note that some hydrodynamic codes will always lead to smooth results no matter what the resolution. This is achieved through the addition of artificially added numerical or user specified damping that severally degrades the accuracy of the code

Time Step Selection

- Important time step selection considerations
  - Time step must be selected such that Courant number < 0.5 to 0.6
  - Time step must be kept this small due to the explicit treatment of select nonlinear terms in the current release version of ADCIRC and related dispersion/propagation characteristics
  - When time step is too large, wiggles may appear in the time dimension that may also drive spatial scale wiggles. These wiggles may be related to
    - Explicit numerical instabilities
    - Folding of the dispersion curve for large Courant number
    - Related phase propagation errors

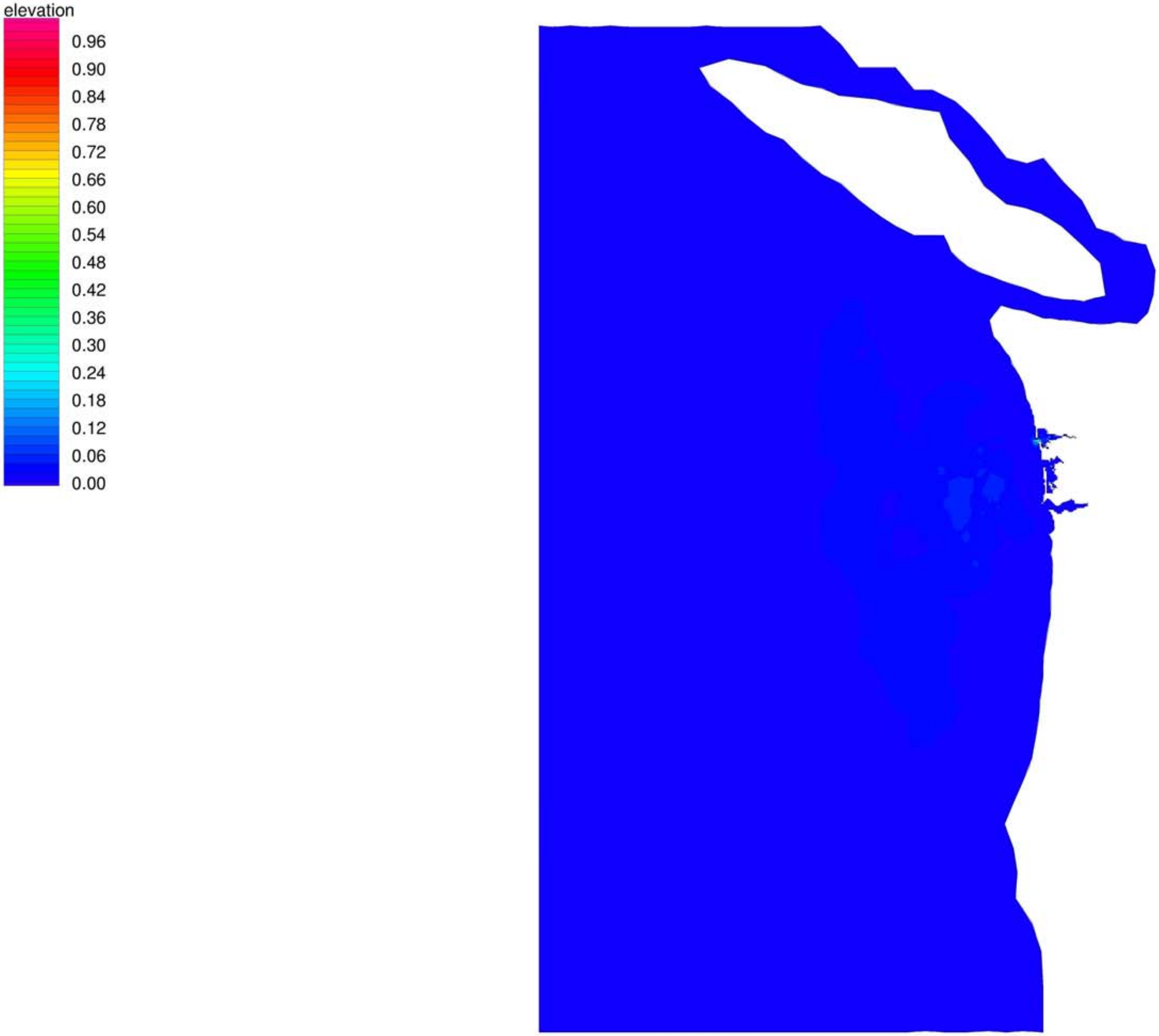
Time Step Selection

- Check Courant Number to establish time step
  - Typically we use a wave celerity based Courant number (that can be applied a priori), although in certain flow regimes it may be useful to check a velocity based Courant number as well

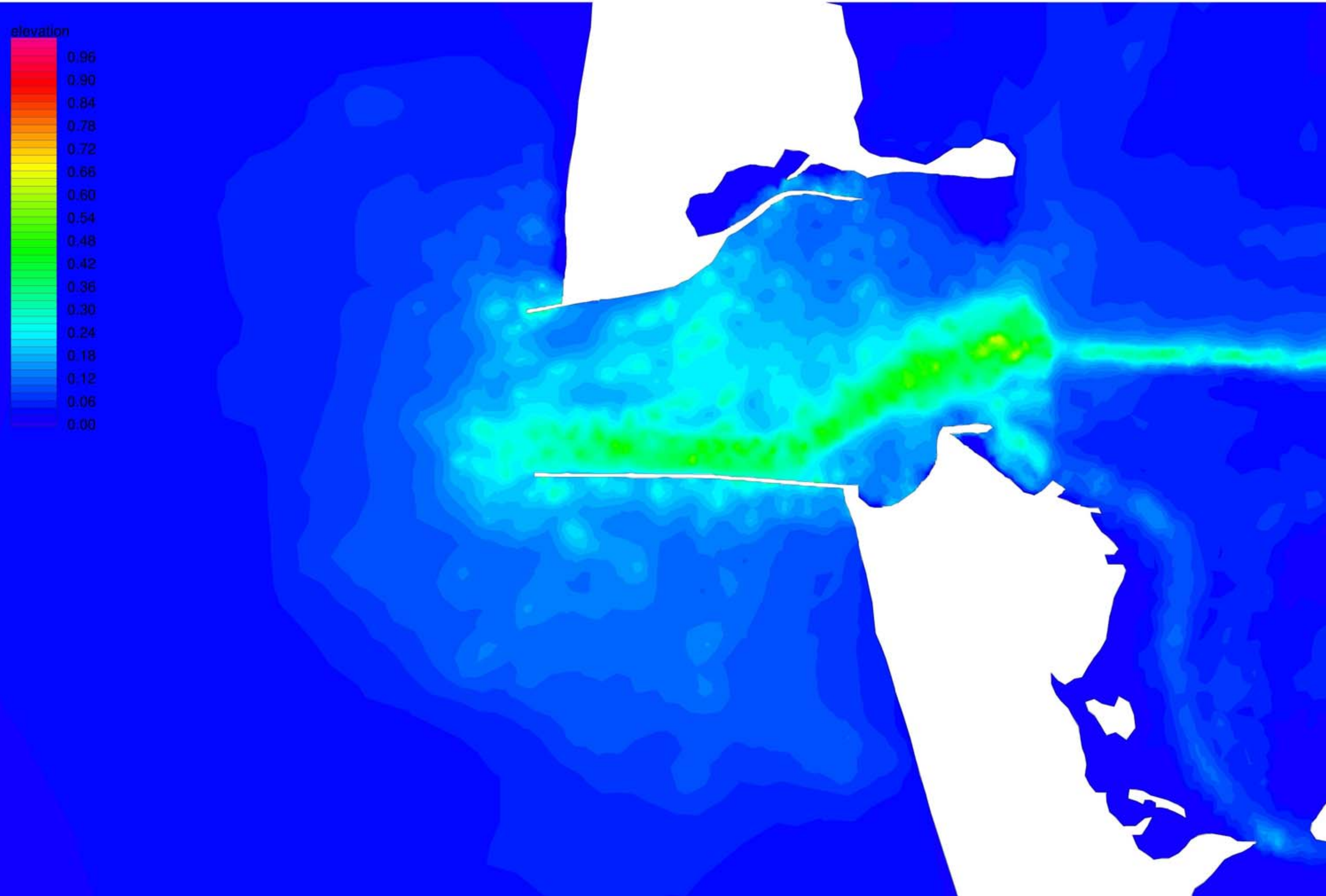
$$C = \frac{\sqrt{gh} \cdot \Delta t}{\Delta x} \quad \text{or} \quad C = \frac{V \cdot \Delta t}{\Delta x}$$

  - Courant number will be higher in small grid size / deep channel regions such as in the inlet channel
  - Time step selected for this case was $\Delta t = 2\,\text{sec}$

Grays Harbor Courant Number



Grays Harbor Courant Number



Time Step Selection

- Time step is controlled by the well resolved deep channel inlet region
- Time step selection in low damped flows (physical or numerical) is very important in order to generate physically correct solutions
- Currently ADCIRC is being Beta tested with a Predictor-Corrector time stepper that allows significantly larger time steps with Courant numbers < 2 to 4 while producing accurate and stable solutions
- Currently under development is a multi time step feature that works in concert with the parallel version of ADCIRC and selects different time steps for different “sub-domains”, dramatically improving overall computational efficiency

GWCE Parameter Selection

- Important considerations regarding the selection of the GWCE weighting parameter, τ_0
- τ_0 is a purely numerical parameter that controls the balance in the GWCE between the “wave equation” portion and the “primitive continuity equation”
- Primitive Continuity Equation (PCE)

$$\frac{\partial \varsigma}{\partial t} + \frac{1}{R\cos\phi}\left(\frac{\partial UH}{\partial \lambda} + \frac{\partial VH\cos\phi}{\partial \phi}\right) = 0$$

GWCE Parameter Selection

- Conservative Momentum Equations (CME)

$$\frac{\partial UH}{\partial t}+\frac{1}{R\cos\phi}\left(\frac{\partial UUH}{\partial\lambda}+\frac{\partial UVH\cos\phi}{\partial\phi}\right)-\left(\frac{U\tan\phi}{R}+f\right)VH=$$

$$-\frac{H}{R\cos\phi}\frac{\partial}{\partial\lambda}\left[\frac{p_s}{\rho_0}+g(\varsigma-\alpha\eta)\right]+M_\lambda+D_\lambda+\frac{\tau_{s\lambda}}{\rho_0}-\frac{\tau_{b\lambda}}{\rho_0}$$

$$\frac{\partial VH}{\partial t}+\frac{1}{R\cos\phi}\left(\frac{\partial VUH}{\partial\lambda}+\frac{\partial VVH\cos\phi}{\partial\phi}\right)-\left(\frac{U\tan\phi}{R}+f\right)UH=$$

$$-\frac{H}{R}\frac{\partial}{\partial\phi}\left[\frac{p_s}{\rho_0}+g(\varsigma-\alpha\eta)\right]+M_\phi+D_\phi+\frac{\tau_{s\phi}}{\rho_0}-\frac{\tau_{b\phi}}{\rho_0}$$

GWCE Parameter Selection

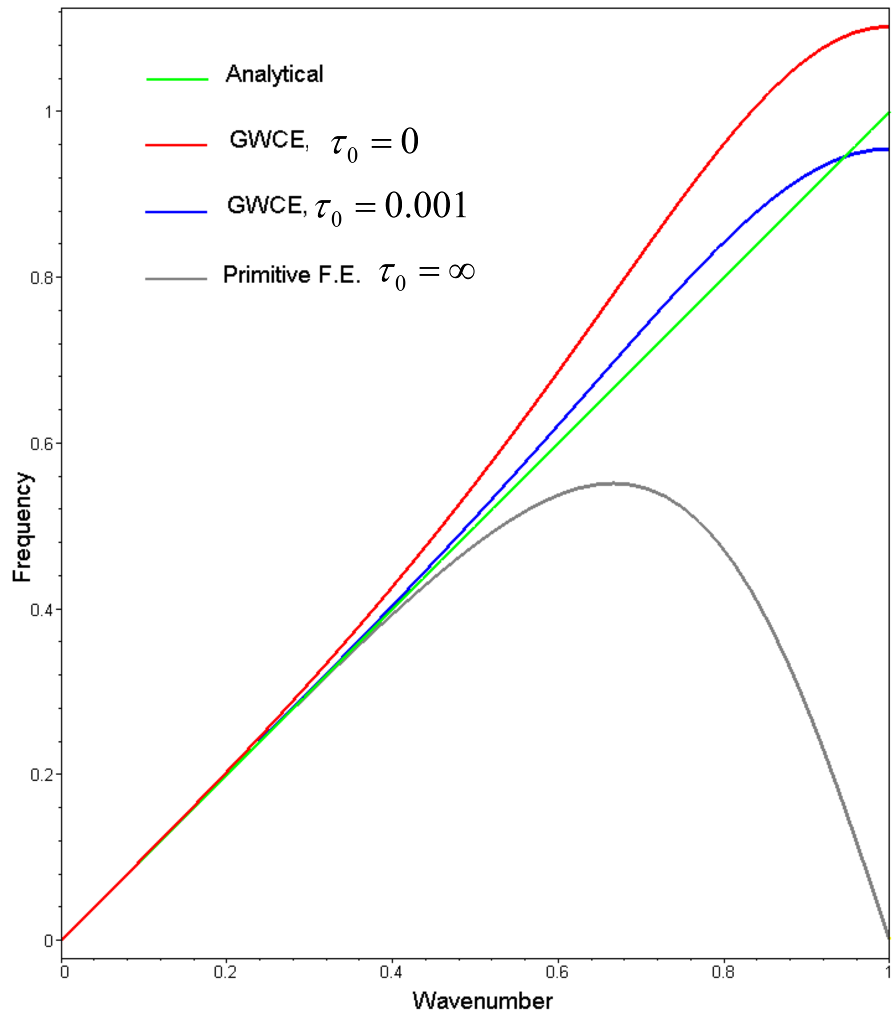
- Generalized Wave Continuity Equation (GWCE)

$$GWCE \equiv \frac{\partial}{\partial t}\left(PCE\right) - \nabla \cdot \left(CME\right) + \tau_0\left(PCE\right) = 0$$

- The full primitive solution leads to a folded dispersion curve
  - Low wavenumber (long wave) physical wave
  - High wavenumber (short wave) spurious wave (noise)

- The use of the GWCE instead of the primitive continuity equation solved in conjunction with the momentum equations, leads to a monotonic dispersion curve
  - Only a low wavenumber (long wave) physical wave
  - Monotonic dispersion relationship prevents the generation of spurious oscillations

GWCE Parameter Selection



GWCE Parameter Selection

- Parameter Ranges
  - τ_0 = 0 $\rightarrow$ Pure Wave Equation
  - τ_0 = infinity $\rightarrow$ Primitive Equation

- Effect of parameter selection
  - τ_0 too low $\rightarrow$ Poor local mass conservation
  - τ_0 too high $\rightarrow$ Folded dispersion curve $\rightarrow$ spurious modes

- Poor mass conservations can be manifested in diverging flows

- Correct selection of τ_0 ***range*** is related to the local frictional balance

GWCE Parameter Selection

- Optimal balance in GWCE based on extensive numerical experiments

$$\tau_0 = (2 \rightarrow 10)\tau_*$$

- Friction term

$$\tau_* = C_f \frac{\left(U^2 + V^2\right)^{\frac{1}{2}}}{H}$$

- Minimum value

$$\tau_0 = 0.001$$

- These values appear to give good mass conservation as well as good dispersion properties

GWCE Parameter Selection

- Based on comparison to TELEMAC's Quassi-Bubble algorithm (which is to the highest order equivalent to the GWCE):

$$\tau_0 = (\frac{\hat{i}\,\omega + 3\tau_*}{2})$$

- User must assess regional friction balance prior to specifying τ_0
- Specify lower τ_0 values in deeper water where flow is slower and depths are greater
- Specify higher τ_0 values in shallower waters where flow is faster and depths shallower

GWCE Parameter Selection

- User must select
  - Spatially constant τ_0 option
  - Spatially variable τ_0 option based on depth alone

$$\text{H} \le 10 \text{ meters}: \qquad \tau_0 = 0.020$$

$$\text{H} > 10 \text{ meters}: \qquad \tau_0 = 0.005$$

  - Spatially variable τ_0 option based on user specified values
  - Spatially variable τ_0 option based on depth and user specified values
- For Grays Harbor, $\tau_0 = 0.005$ works very well
  - Good mass conservation
  - Smooth flows
  - Results not sensitive to changing value to $\tau_0 = 0.01$

Summary

- A computational solution is only as good as
  - The specified geometry and bathymetry
  - The spatial and temporal resolution
  - The specification of physically correct model parameters
  - The specification of accurate and physically well balanced boundary conditions and forcing functions

- It is necessary that the computational model
  - Be inherently non-dissipative
  - Be accurate
  - Be efficient and able to apply fine grids in space and time

Summary

- If wiggles appear, check
  - Spatial resolution
  - Temporal resolution
  - Specification of numerical parameters such as
    - GWCE parameter τ_0
    - Time weighting parameters

- Don’t suppress the wiggles, they’re telling you something!!

Summary

- The modeler should check the integrity and accuracy of the calculation by
  - Doing a grid convergence study
    - Halve space and time steps and check solutions
    - Apply Richardson extrapolation to estimate errors (Westerink and Roache, 1997)
  - Ensure that model parameters do not affect the solution beyond estimated gridding errors