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ANALYTICAL MODEL OF NAVIGATION CHANNEL INFILLING BY CROSS-CHANNEL TRANSPORT

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Abstract: An analytic model is presented for estimating the time-evolution of bank encroachment, sediment deposition, and bypassing of a channel of specified initial cross section exposed to an active zone of longshore or other known cross-channel sediment transport. It is applicable to channels in estuaries, bays, and lakes by input of the rate of sediment transport (sand to gravel range) approaching normal to the channel. The model can be applied to estimate necessary depth and width of a channel to be newly dredged or the performance of a channel to be deepened and widened. The model is based on the continuity equation governing conservation of sediment volume, together with typically available or estimated input transport rates in engineering applications. An analytical solution of the linearized coupled differential equations has pedagogic value and gives insight into the processes of channel infilling and bypassing. The analytic model can also serve as an engineering screening tool. Numerical solution of the full nonlinear, coupled equations allows extension to more complex situations.

INTRODUCTION

Navigation channels issuing through inlet entrances intercept sediment moving alongshore. Sediment transported to a navigation channel can reduce channel width by accumulating on the sides (bank encroachment), and channel depth can be reduced through deposition on the bottom (Fig. 1). Sediment can also pass over a channel by moving in suspension, and material deposited in the channel can be re-suspended and transported out, both processes contributing to bypassing. At inlets, the bypassing rate in the predominant direction of transport to the down-drift beach enters in sediment budgets, whereas the gross rate of longshore transport relates to channel dredging requirements. In the present discussion, along-channel transport is omitted, as is bi-directional transport, although the latter is readily accommodated in the present model framework. Numerical solutions can accommodate these processes, whereas analytic solutions are sought here.

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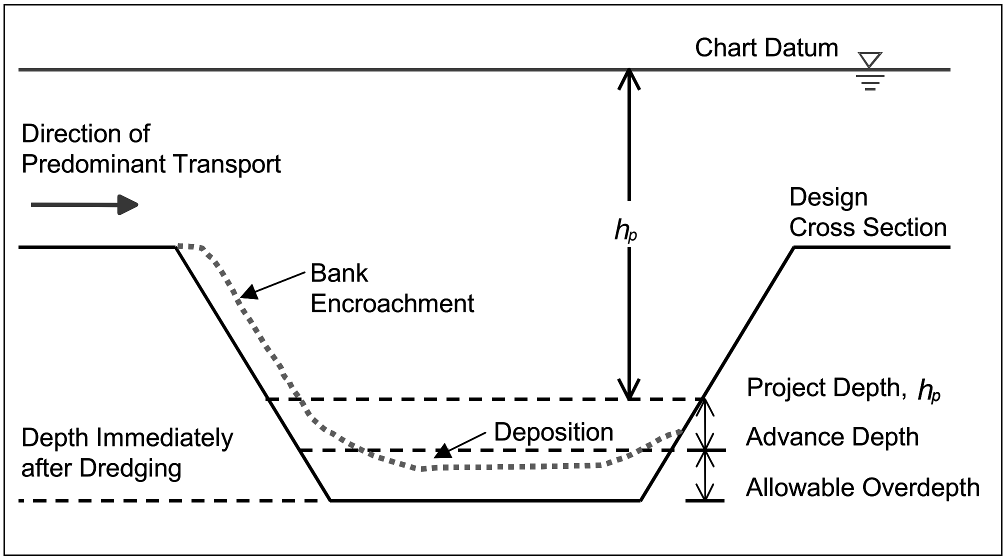

Fig. 1. Schematic of channel infilling by encroachment and deposition

Referring to Fig. 1, navigation can be limited by bank encroachment, typically as intrusion of a spit, shoal, or sand wave into a channel, or through deposition of sediment along the bottom of a channel. If some minimum width is reached or if the channel bottom grows above project depth, maintenance dredging is necessary. Dredging requirements must be estimated in the design and modification of channels. Deepening and widening may be done under a plan of advance maintenance to reduce dredging frequency, cost, and environmental impact. Savings can be accrued by reducing the number of mobilizations, demobilizations, and channel-condition surveys, or by scheduling dredges when equipment can be shared among projects. Advance maintenance may also be considered to take advantage of favorable weather windows, either to reduce the cost of dredging or to maintain the channel through storm seasons when maintenance is more expensive or risky.

Several procedures have been proposed for calculating channel deposition, and only a few are mentioned here. Trawle (1981) developed an empirical method based on dredging data for a site. Simplified calculation methods have be given by Gole and Tarapore (1971) and Galvin (1982). Of these, the Galvin (1982) procedure depends on readily available information and accounts for an along-channel current. Foreman and Vallianos (1984) applied the Galvin (1982) method to estimate performance of a channel at Oregon Inlet, North Carolina, USA. More sophisticated models have been developed that represent the acting micro-scale processes, including the vertical profiles of flow and sediment concentration, as well as flow separation (Kadib 1976; Van Rijn 1991; Walstra et al. 1999; van de Kreeke et al. 2002). Despite the availability of these and other calculation procedures, a clear and simple mathematical description of channel infilling and bypassing appears to be lacking. In addition, a procedure is needed that represents bank encroachment as well as sediment deposition.

This paper introduces a simple morphologic mathematical model that describes the basic processes of channel bank encroachment and channel deposition as produced by cross-channel sediment transport. An analytical solution of the model equations obtained under reasonable assumptions reveals physical dependencies of channel behavior. The full model is readily extended to more complex situations by numerical solution.

ASSUMPTIONS

Channel infilling can occur through an arbitrary combination of bedload transport, which decreases channel width; and through suspended load transport, which decreases channel depth. Bypassing can be represented by suspended load passing over the channel and by re-suspension and transport of material that has been deposited in the channel. Such processes are depicted schematically in Fig. 2. Assumptions underlying the model are:

1. Infilling by bedload can create a shoal at the edge of the channel and thereby constrict the channel (bank encroachment). Encroachment decreases channel width.

2. Sediment can be deposited directly into the channel.

3. The slope of the channel remains constant. (After dredging, slumping may occur to achieve the angle of repose, and this process is neglected.)

4. The channel does not erode on the down-drift side. (This assumption will be removed in a future version of the model.)

5. Channel slopes are sufficiently mild that flow separation and secondary circulation (which can cause sediment near the bed to move against the upstream flow direction) do not occur or can be neglected.

6. Sediment transport along the channel, as by tidal action or a river current, is negligible or has constant along-channel gradient at the cross-section of interest. (Transport by ebb and flood currents along the channel will be introduced in a future version of the model.)

7. Cross-channel (longshore) transport is predominantly unidirectional. (This assumption is easily eliminated in numerical solution of the model.)

8. Material that is deposited in the channel can be resuspended and leave the channel, and the rate of resuspension is proportional to the depth in the channel at that time and the rate of deposition.

Figure 2 illustrates the conceptual framework of the model for the situation of transport directed to the right, assumed to be the dominant direction of transport. A numerical version of the model can readily treat both left- and right-directed transport. Immediately after dredging, the channel has width W_0 and depth h_0. The ambient or natural depth in the vicinity of the channel is h_a. As sediment is transported to the channel, it can become narrower by filling from the side and shallower by filling from the bottom. The coordinate z measures elevation from the bottom of the dredged channel. It is convenient to work with elevation from the dredged bottom rather than depth; conversion to depth below the navigation datum can then be made through knowledge of z, h_0, and h_p, the project depth.

If the channel becomes narrower because of growth of the updrift side by bedload transport and deposition into the channel, the width of the channel at a given time is:

$$W(x,t) = W_0 - x(t), \qquad \text{for } x < W_0 \tag{1}$$

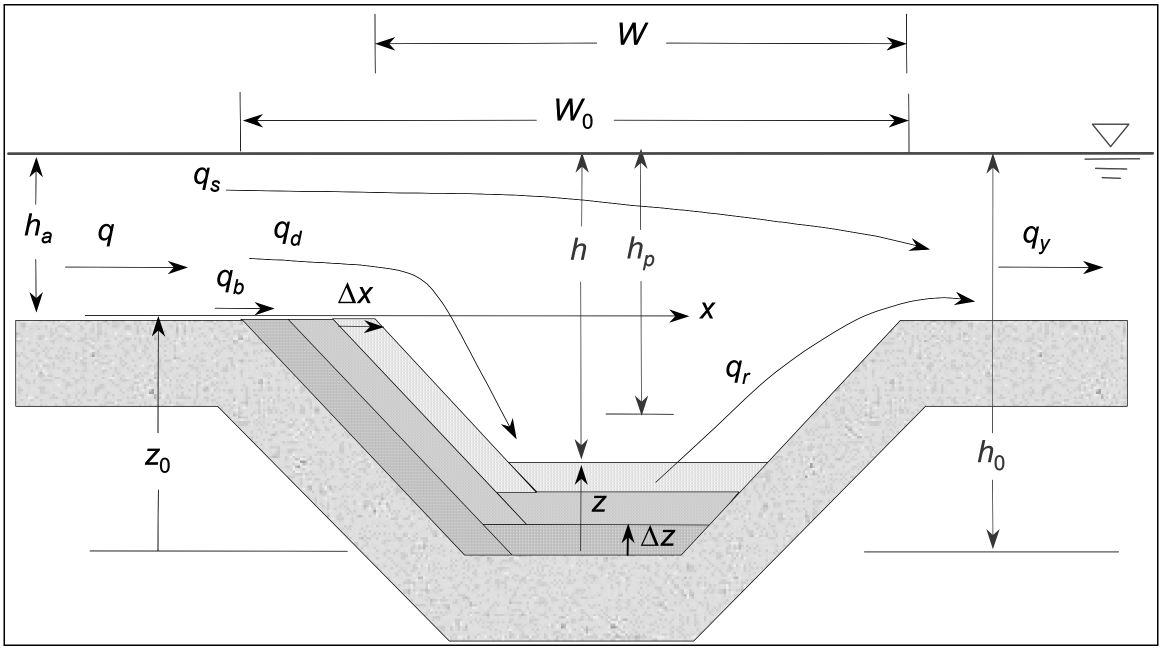

Fig. 2. Definition sketch for channel infilling model

The transport rate q per unit length of channel near the updrift side of the channel can be divided into a bedload transport rate q_b, a rate of suspended material deposited into the channel q_d , and a rate q_s of suspended material passing over the channel. For the situation of a coastal inlet, for the portion of channel crossing the surf zone, the transport rate per unit channel length can be estimated as the total transport rate Q multiplied by the ratio of length of channel exposed to the longshore transport to the total width of the surf zone.

From Fig. 2 and above discussion, the transport rate per unit length of channel is represented as:

$$q = q_b + q_d + q_s \tag{2}$$

The rate q_c of material filling the channel is the sum of that entering and that resuspended and leaving the channel:

$$q_c = q_b + q_d - q_r \tag{3}$$

The rate q_y of material bypassing the channel is given by the sum of q_s and the rate q_r of material resuspended from the channel:

$$q_y = q_s + q_r \tag{4}$$

It is seen that $q = q_c + q_y \; (= q_b + q_d + q_s)$.

A closure assumption of the model is that the rate of material resuspended from the channel bottom is proportional to the product of the depth of the channel (normalized by the total channel depth) and the rate of deposition as,

$$q_r = \frac{z}{z_0} q_d \tag{5}$$

which is equivalent to the closure assumption invoked in the coastal inlet Reservoir Model (Kraus 2000).

To proceed, the apportionment of q must be known. For this purpose, partitioning coefficients a are introduced, where the a's are numbers or, more generally, functions of the ambient conditions (which can be expressed as decimal fractions of unity or percentages). A subscript denotes the association or coupling to the input transport rate. Thus,

$$\begin{aligned} q_b &= a_b q \\ q_d &= a_d q \\ q_s &= a_s q \end{aligned} \tag{6}$$

These coefficients obey the constraint:

$$a_b + a_d + a_s = 1 \tag{7}$$

The constraint expresses one equation in three unknowns, requiring two additional equations. To proceed, in the absence of process-based estimates, one can, for example, specify a_b and a_d as inputs and solve for a_s as $a_s = 1 - a_b - a_d$. The determination of the coupling coefficients in terms of the time dependent coastal processes at the site is the subject of future work. At the moment, values are specified based on experience gained with the model (see the examples below).

For the channel bottom, the continuity equation gives a change in bottom elevation Δz in time interval Δt as,

$$(W_0 - x)\Delta z = \left(q_d - q_r\right)\Delta t = \left(q_d - \frac{z}{z_0} q_d\right)\Delta t = a_d q \left(1 - \frac{z}{z_0}\right)\Delta t$$

which becomes:

$$\frac{dz}{dt} = \frac{a_d}{W_0 - x}\left(1 - \frac{z}{z_0}\right) q, \qquad z(0) = 0 \tag{8}$$

Similarly, for infilling by growth of the side channel, continuity gives,

$$\Delta x\left(z_0 - z\right) = q_b \Delta t = a_b q \Delta t$$

which becomes:

$$\frac{dx}{dt} = \frac{a_b}{z_0 - z} q, \qquad x(0) = 0 \tag{9}$$

Equations 8 and 9 are simultaneous non-linear equations for channel depth z and width x as a function of the input rate (which can be time dependent) and time. Equation 8 indicates that z will increase more rapidly as the width decreases, and Eq. 9 indicates that the width $W(x)$ will decrease more rapidly as the channel fills. These equations can be solved numerically for a general situation with time-dependent variables. An analytic solution approach for rapid desk study is given next. The analytic solution reveals physical dependencies and yields several simple expressions governing channel performance.

ANALYTICAL SOLUTION FOR CHANNEL INFILLING

Ongoing maintenance of channels will not allow the depth to become less than project depth or allow the width of the channel to be greatly reduced. These conditions are equivalent to stating mathematically that practical applications concern a relatively short time interval after dredging as compared to the total time required to fill a channel completely. For this situation, the equations can be linearized under the reasonable assumptions $z/z_0 << 1$ and $x/W_0 << 1$. By expansion of denominators, Eqs. 8 and 9 become,

$$\frac{dz}{dt} = \frac{a_d}{W_0} q \left(1 - \frac{z}{z_0} + \frac{x}{W_0}\right) \qquad z(0) = 0 \tag{10}$$

and

$$\frac{dx}{dt} = \frac{a_b}{z_0} q \left(1 + \frac{z}{z_0}\right) \qquad x(0) = 0 \tag{11}$$

which are now simultaneous linear equations for z and x.

Differentiating Eq. 10 with respect to time and substituting Eq. 11 into the resultant equation to replace dx/dt gives,

$$\frac{d^2 z}{dt^2} + 2b\frac{dz}{dt} - cz = d, \qquad z(0) = 0, \quad z'(0) = \frac{a_d}{W_0} q \tag{12}$$

where the quantities b, c, and d are defined as:

$$b = \frac{a_d}{2\, W_0 z_0} q, \qquad c = \frac{a_b a_d}{W_0^2 z_0^2} q^2, \qquad d = cz_0 \tag{13}$$

A second initial condition for z was introduced through the first derivative as determined from Eq. 8 evaluated with the initial conditions on x and z. The solution of Eq. 12 is found to be,

$$z = C_1 \exp(r_1 t) + C_2 \exp(r_2 t) - z_0 \tag{14}$$

where

$$r_1 = -b + \sqrt{b^2 + c}, \qquad r_2 = -b - \sqrt{b^2 + c} \tag{15}$$

and

$$C_1 = \frac{z'(0) - r_2 z_0}{r_1 - r_2}, \qquad C_2 = -C_1 + z_0 \tag{16}$$

It can be seen from Eqs. 14 and 15 that this solution is valid for relatively short times after $t = 0$ because the term proportional to $\exp(r_1 t)$ diverges for long elapsed time ($r_1 > 0$).

Substituting Eq. 14 into Eq. 9 and integrating gives

$$x = \frac{a_b}{z_0^2} q_R \left[\frac{C_1}{r_1} \left(\exp(r_1 t) - 1 \right) + \frac{C_2}{r_2} \left(\exp(r_2 t) - 1 \right) \right] \tag{17}$$

Various simplified explicit expressions can be obtained for engineering quantities of interest through limiting forms of the analytic solution for relatively short elapsed times after dredging, as given next.

For small t, Eq. 14 can be expanded to give (retaining leading order in t, a_d, and a_b),

$$z = \frac{a_d}{W_0} qt - \frac{a_d}{2W_0^2 z_0} \left(a_d + a_b \right) q^2 t^2 \tag{18}$$

indicating that the channel starts filling linearly with time. If $a_b = 0$ (no bedload transport), then $x = 0$ for all time, and Eq. 14 reduces to,

$$z = z_0 \left[1 - \exp\left(\frac{-a_d}{W_0 z_0} q\, t \right) \right] \tag{19}$$

which indicates exponential filling of the channel.

Similarly, for small t, Eq. 17 yields,

$$x = \frac{a_b}{z_0} qt + \frac{a_b a_d}{2 W_0 z_0^2} q^2 t^2 \tag{20}$$

showing that the channel fills linearly manner with time by intrusion of the updrift side into the channel and that deposition by suspended material is governed by a lower order quadratic dependence in time for relatively short elapsed time after dredging.

Channel Infilling Rate: The rate of channel infilling, the rate at which the bottom shoals, is $R_z = dz/dt$ and can be calculated from Eq. 18. For a relatively short time after dredging:

$$R_z = \frac{a_d}{W_0} q - \frac{a_d}{W_0^2 z_0}\left(a_d + a_b\right) q^2 t \tag{21}$$

The leading-order term is independent of z_0, so the rate of channel infilling depends more strongly on W_0 than on z_0. The solution indicates that the rate of channel infilling can be reduced more by increasing channel width than by increasing channel depth.

Bypassing Rate: The bypassing rate, $q_y = q_s + q_r$, becomes,

$$q_y = (1 - a_d + a_d \frac{z}{z_0}) q \tag{22}$$

which is a function of time through z (Eq. 14).

Time Interval for Maintenance Dredging: A channel section is dredged to a design depth including a certain amount of advance dredging and a certain amount of allowable overdredging. The analytical channel infilling model provides an explicit expression for estimating of the maximum possible time interval Δt_p between dredging events (the dredging cycle) for a constant rate of infilling. Then, for an increase in channel elevation from initial depth h_0 (elevation $z = 0$) to some the project depth h_p (or elevation $z_p = h_0 - h_p$), at which time dredging must be scheduled, Δt_p can be determined from Eq. 14 by iteration.

If bedload transport (channel bank encroachment) is not significant, then Eq. 19 can be solved to give:

$$\Delta t_p = -\frac{W_0 z_0}{a_d q} \ln\left(1 - \frac{z_p}{z_0}\right) = -\frac{W_0 (h_0 - h_a)}{a_d q} \ln\left(\frac{h_p - h_a}{h_0 - h_a}\right) \tag{23}$$

This equation indicates that the time between dredging intervals is directly proportional to the width of the channel; approximately proportional to the initial depth of the channel with respect to the ambient depth; and inversely proportional to the input transport rate. If Eq. 23 is expanded or, equivalently, Eq. 18 is solved for Δt_p to leading order, the result is:

$$\Delta t_p \cong \frac{W_0}{a_d q}\left(h_0 - h_p\right) \tag{24}$$

EXAMPLE SOLUTIONS

In the two examples to follow, $z_0 = 4$ m, $W_0 = 50$ m, and time step $\Delta t = 0.1$ year. The effective channel length, determined as the average width of the surf zone over all tides and wave conditions, is estimated to be 1,000 m. Equations 8 and 9 (simultaneous non-linear equations) were solved numerically, and the analytical model developed from the linearization (Eqs. 14 and 17) was also run. The simulation time was 2 years, and the numerical calculation was halted if z reached $z_0/2$ (assumed project depth) or x reached $W_0/2$ (minimum allowable width of channel).

Example 1: (fine sand) *Q* = 150,000 m³/year, a_d = 0.5, a_b = 0.1.

The sediment at this site is fine sand, and experience with sensitivity testing of the model and comparison to limited data gives $a_d = 0.5$, with little bedload ($a_b = 0.1$). Larson and Kraus (2001) give a procedure for estimating a_d consistent with the present simplified approach. This example simulates a shallow-draft channel at an inlet located on a sandy shore, so most of the sand is deposited into the channel or passes over the channel ($a_s = 1 - a_d - a_b = 0.4$). If material is deposited into the channel, it can be readily resuspended. The effective channel length is 1,000 m, so q = 150,000/1,000 = 150 m³/m/year.

Figures 3 and 4 compare calculations with the numerical model and the linearized model. For short elapsed time there is agreement, with deviations occurring after about 0.6 to 0.8 year for this example. After 1.7 years, project depth ($z = z_0$) was reached, and the numerical model stopped.

Figure 5 shows the time evolution of the channel infilling rate q_c and the bypassing rate q_y normalized by the input q. The total adds to unity at any given time. The rate of bypassing exceeds the channel infilling rate approximately 0.6 years after dredging.

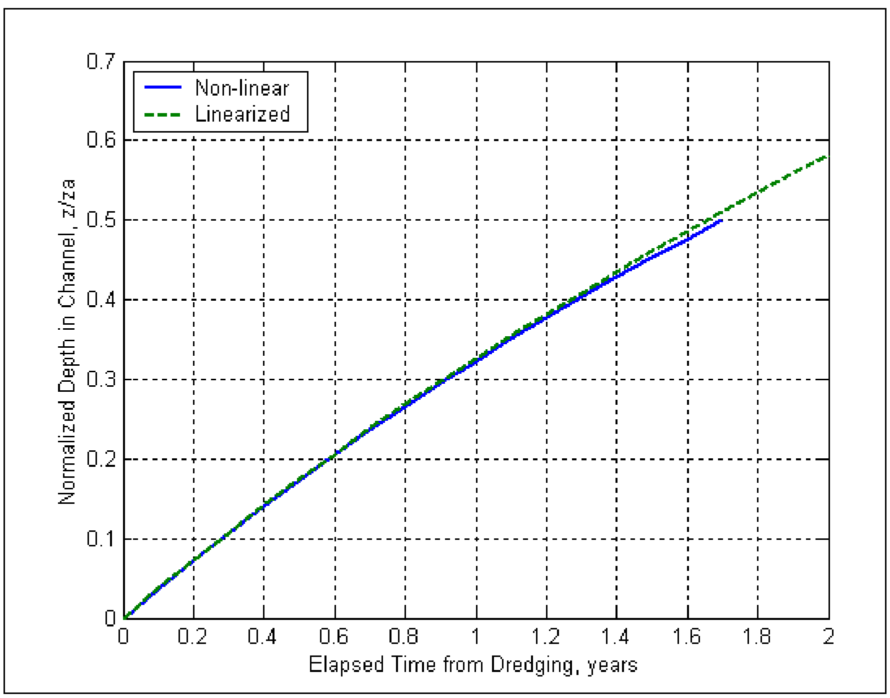

Fig. 3. Increase in elevation (decrease in depth) in channel on sand shore: comparison of non-linear model (numerical solution) and linearized model (analytical solution)

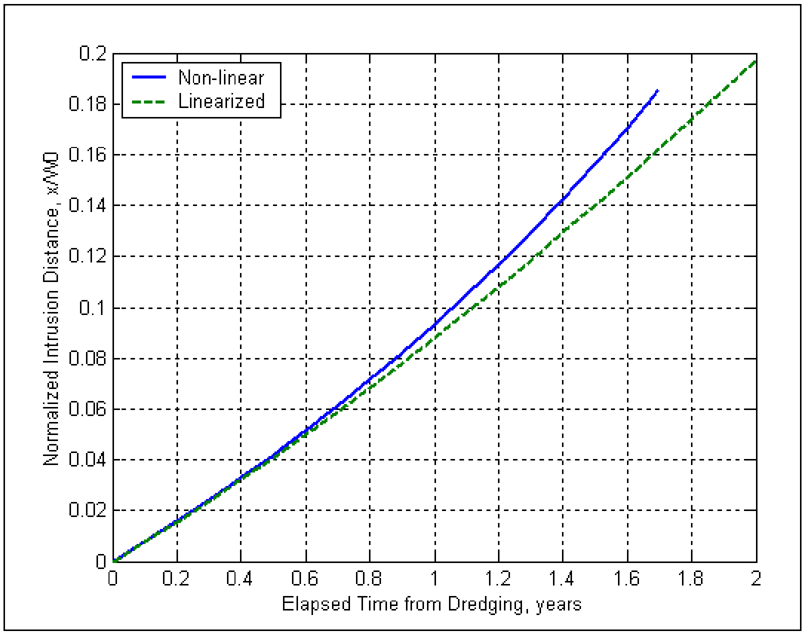

Fig. 4. Increase in intrusion distance (decrease in width) in channel on sand shore: comparison of non-linear model (numerical solution) and linearized model (analytical solution)

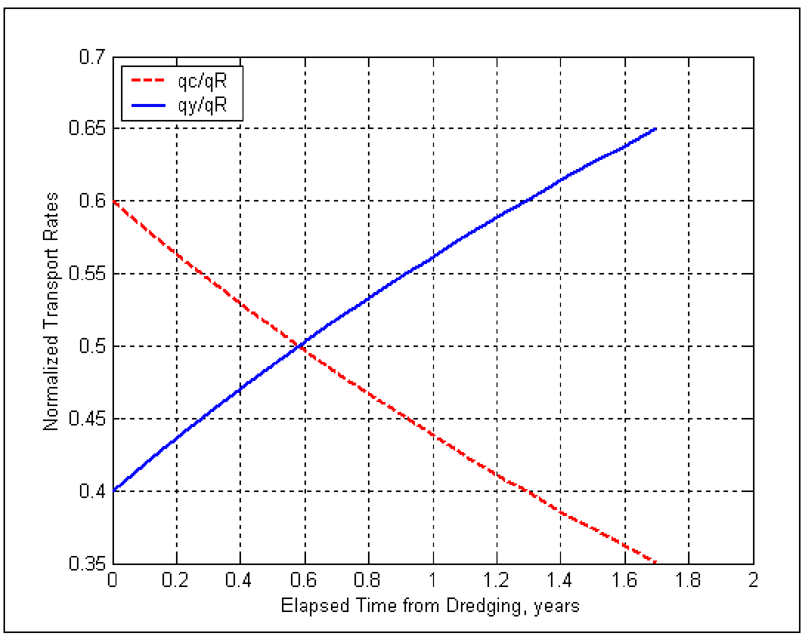

Fig. 5. Evolution of channel infilling rate and channel bypassing rate on sand shore

Example 2: (gravel) Q = 50,000 m^3/year, a_d = 0.3, a_b = 0.6

This example simulates a shallow-draft channel in an inlet located on a gravel shore, so most of the coarse-grained material is deposited on the updrift side of the channel, with little bypassing by suspended transport ($a_s = 1 - a_d - a_b = 0.1$). Only the fine material, a limited amount, is assumed to travel over the channel by suspension. If the gravel falls into the channel, none is resuspended sufficiently to leave it. The effective channel length is 1,000 m, so q = 50,000/1,000 = 50 m^3/m/year.

In this situation of a gravel shore, because most material remains at the updrift side of the channel, little depth is lost (Fig. 6). However, after 2 years, the updrift side of the channel has intruded about 37 percent of the way across the channel (Fig. 7), becoming a hazard to navigation. The side of the channel grows approximately linearly, because little material is deposited in the channel bottom through suspension. Therefore, the governing equation is only weakly nonlinear, and the linearized (analytical) solution and numerical solution produce almost the same results. Figure 8 shows that most of the material is deposited into the channel, with little bypassing.

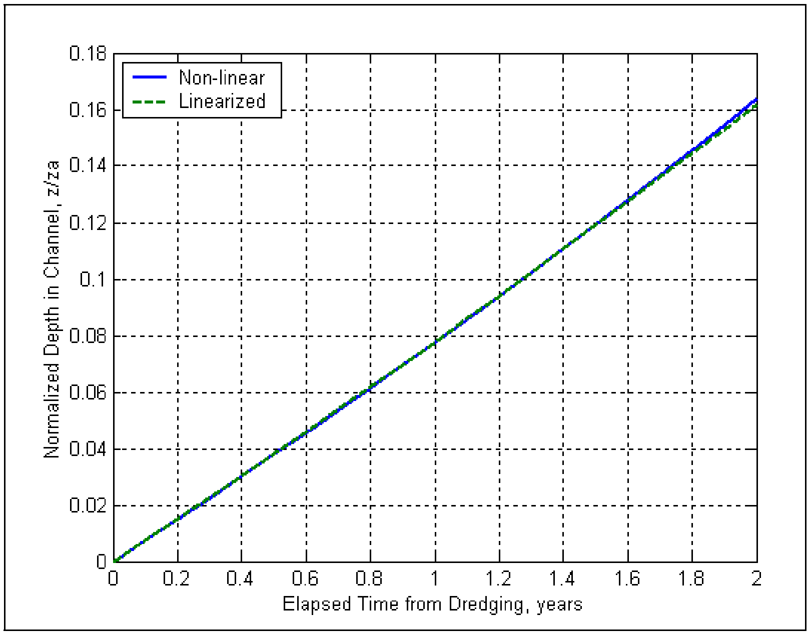

Fig. 6. Increase in elevation (decrease in depth) in channel on gravel shore: comparison of non-linear model (numerical solution) and linearized model (analytical solution)

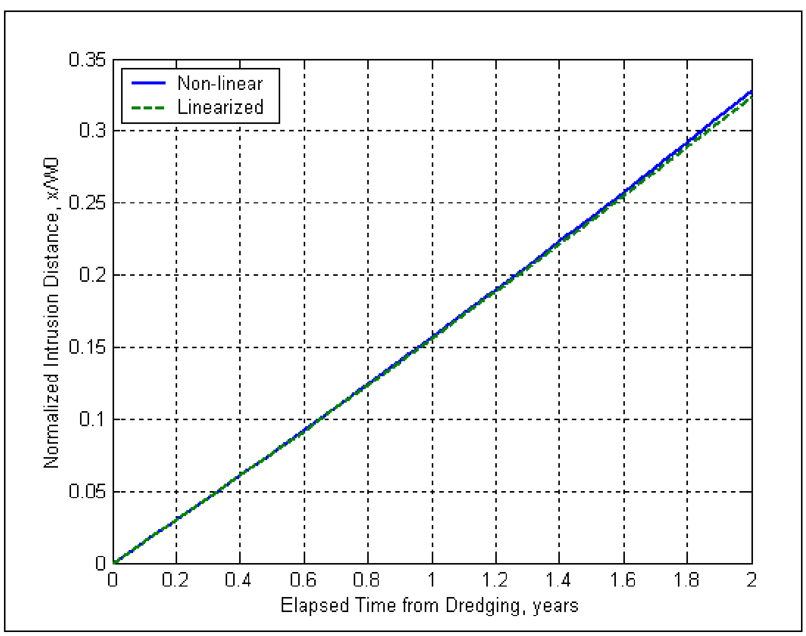

Fig. 7. Increase in intrusion distance (decrease in width) in channel on gravel shore: comparison of non-linear model (numerical solution) and linearized model (analytical solution)

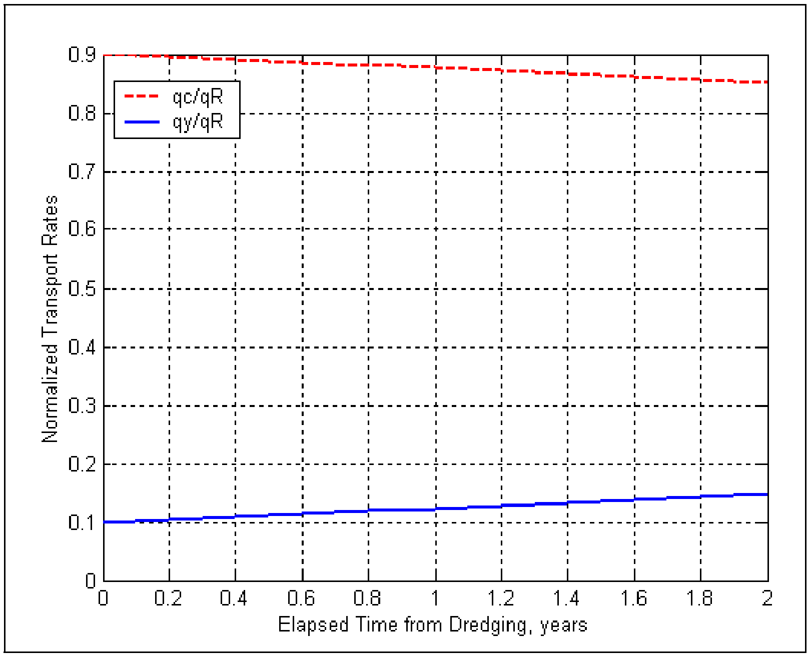

Fig. 8. Evolution of channel infilling rate and channel bypassing rate on gravel shore

CONCLUDING DISCUSSION

A simple mathematical model of channel bank encroachment, infilling, and bypassing by cross-channel sediment transport has been developed. The model represents a substantial amount of the acting physical processes in a schematic way, producing a set of two coupled, non-linear first-order differential equations. By making an assumption compatible with the reality of channel maintenance (i.e., the channel will never be allowed to fully close), a linear set of coupled equations is obtained that can be solved. The solution explicitly reveals factors representing channel performance in terms of the governing physical dependencies of channel geometry, upstream transport rate, and partition of transport as bed load or suspended load.

In addition of serving as a possible screening tool to quickly and conveniently assess alternative channel designs, the simplicity of the model in representing fairly complex physical processes holds pedagogic value for explaining channel sediment processes. Work is underway in the Coastal Inlets Research Program to provide a convenient interface for implementing the numerical solution of the channel infilling model. The solution will allow time-dependent wave information to generate a longshore current, calculate the width of the surf zone, and channel infilling by sections with different ambient depths along the channel. Further research in micro-scale processes will allow expression of the partitioning coefficients (a's) in terms of ambient forcing conditions and channel geometry.

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