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EQUILIBRIUM SHAPE OF HEADLAND-BAY BEACHES FOR ENGINEERING DESIGN

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Abstract: The equilibrium shoreline form of crenulate or headland-bay beaches was identified in the 1940s and is widely accepted by coastal geomorphologists and engineers. However, little quantitative verification of the standard functional shoreline forms, the logarithmic-spiral shape and parabolic shape, has been made. In addition, limited guidance is available for applying the functional shapes in engineering practice. In this paper, we investigate the two shapes by fitting to 46 beaches in Spain and in the North America covering from large regional scale to small project scale. Software is described which automates the fitting. A new function, called the hyperbolic-tangent shape, is introduced for engineering applications. The hyperbolic-tangent shape is easy to fit, and its controlling parameters have simple geometric interpretation. Guidance is given for interpreting and fitting the three headland-bay beach functions, with background data listed.

INTRODUCTION AND BACKGROUND

The concept of an equilibrium shape of crenulate or headland-bay beaches was introduced by Krumbein (1944), followed by the work of Silvester (1960) for engineering applications and by Yasso (1965) for fitting of a logarithmic spiral to data. Headland control, leading to a crenulate-shaped equilibrium beach planform, has been advocated for engineering use by Silvester and Ho (1972) and Silvester and Hsu (1993) as a naturally functioning and preferable means of shore protection.

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Because little quantitative verification of proposed headland-bay shapes could be found, one objective of the present study is to investigate the generality of these equilibrium shapes through analysis of a large data set of beach response to headlands ranging from regional scale to local engineering projects. The second objective is to develop guidance for applying the idealized shapes in engineering and morphology studies. In the process of investigating the shapes and developing engineering guidelines, a new shape was developed which we call the hyperbolic-tangent shape.

Previous researchers proposed two functions for describing the equilibrium shoreline of a headland-bay beach, the logarithmic spiral (op. cit.) and the parabolic shape (e.g., Hsu et al. 1987; Hsu and Evans 1989; Silvester and Hsu 1993). None of these shapes, including the hyperbolic-tangent shape proposed in this study, is derived directly from the acting physical processes that developed the shape; rather, they are observational. The empirical approach appears strong qualitatively, because of the existence of the many headland beaches found on all coasts of the world. The weakness of the approach lies in the quantification process of fitting a shape to data and for design, where data will not exist, requiring the engineer to exercise judgement in the fitting procedure. Many more references were consulted and reviewed than can be given here. We plan to discuss these, the assembled database, and calculation algorithms in another publication.

As discussed in the original sources, headland-bay equilibrium shoreline shapes arise through wave sheltering by diffraction at the object serving as the headland, combined with refraction, which will dominate with distance along the beach away from the headland. This explanation tends to require the condition of a strongly predominant wave direction. LeBlond (1972) studied the existence and properties of a planimetric shape towards which a headland beach asymptotically approached. He also attempted a numerical simulation of the erosion of a linear beach in the presence of a headland, but was not fully successful. Komar and Rea (1975) and Walton (1977) investigated cause and effect in formation of headland beaches. The problem has yet to be resolved fully, although modern modeling technology appears capable of doing so. Wind (1994) presented an analytical model of crenulate-shaped beach development, for which the beach shape remains constant with time, but with the entire form expanding at a rate according to a time function.

A logarithmic-spiral (hereafter abbreviated as log-spiral) shape eventually turns around the headland and, at some ambiguously determined point whose location depends on the site, it loses meaning for describing shoreline position. The question as to where this cutoff should be is problematic because of limitations of a static (equilibrium) form. Site-specific constraints such as presence of other headlands or sediment-impounding features, trend of bathymetric and topographic contours, and underlying geologic structure exert controls that cause deviations of the shoreline from a simple and smooth form. Sometimes, the turning of the log spiral fits the shape of the shoreline produced by impoundment at a headland-type feature located down drift. In summary, the log-spiral shape might best be viewed as applicable to the beach located between two headlands for a coast with predominant wave direction.

The parabolic shape was developed by Hsu et al. (1987, 1989a, 1989c), Hsu and Evans (1989), and Silvester and Hsu (1991, 1993) to improve agreement, as compared with the log spiral, along the down-drift section of a headland-bay beach. This section is not typically strongly curved distant from the headland, unless it intersects a sediment-impounding structure or headland. In studying single-headland beaches, the present authors developed the hyperbolic-tangent shape as described in this paper, which appears to have advantages over the parabolic shape in ease of application and interpretation of empirical parameters defining it.

In the course of this investigation, automated shape fitting routines were developed in the Graphical User Interface of the MatLab (Version 5) language. These convenient programs are available from the authors upon request.

HEADLAND-BAY DATABASE

A database was developed comprised of 23 beaches each in Spain and in North America. Data were sought for beaches extending from relatively small-project scale (hundreds of meters) to regional scale (exceeding tens of kilometers for the case of the headland bay downdrift of Cape Canaveral). The database and resultant fitting parameters are summarized in tabular form at the end of this paper. The observed headland-bay beaches were classified as having one, two, or “1.5” headlands (partial headland located down drift of the main headland). The database was developed from nautical charts, project drawings, and aerial photographs, from which the shoreline position and the location and geometry of the headland were digitized.

The expansive data set covers a wide range of beach lengths, sediment size from fine to medium sand, types and sizes of headland controls, and incident wave conditions. It therefore provides a challenge to the concept of an equilibrium shape for headland-bay beaches, by which the validity and universality of the shapes can be examined.

LOGARITHMIC SPIRAL SHAPE

Krumbein (1944) observed that a headland-bay beach adopts an equilibrium shape that is similar to a log spiral. The pole of the spiral is identified as the diffraction point (Silvester 1960, 1976), and the characteristic angle of the spiral is a function of the incident wave angle with respect to a reference line. For headlands of irregular shape and for those with submerged sections, the diffraction point cannot be specified unambiguously, a problem entering specification of all equilibrium shapes. The reference line extends from the approximate location of the diffraction point to a downdrift headland. In fitting the log-spiral to data, the location for of the pole can be considered as free parameter to be determined in a best-fit search. In design, some ambiguity exists as to where to locate the pole.

The logarithmic, equiangular, or logistic spiral as shown in Fig. 1 was described by Descartes as the curve that cuts radius vectors from a fixed point O under a constant angle α. It is expressed mathematically in polar coordinates by

$$R = R_0 \, e^{\theta \cot \alpha} \tag{1}$$

where R = length of the radius vector for a point P measured from the pole O; θ = angle from an arbitrary origin of angle measurement to the radius vector of the point P; R_0 = length of radius to arbitrary origin of angle measurement; and α = characteristic constant angle between the tangent to the curve and radius at any point along the spiral.

A property of the log-spiral curve is that the angle a between the tangent to the curve and the vector radius at any point along the curve is constant. This leads to the interesting result that the shape of the log spiral is controlled only by α, with the parameter R_o determining the scale of the shape. In fact, the functioning of R_o is equivalent to setting a different origin of measurement of the angle θ. In other words, graphically the log-spiral may be scaled up or down by turning the shape around its pole. Fig. 2 shows how different values of α alter the shape.

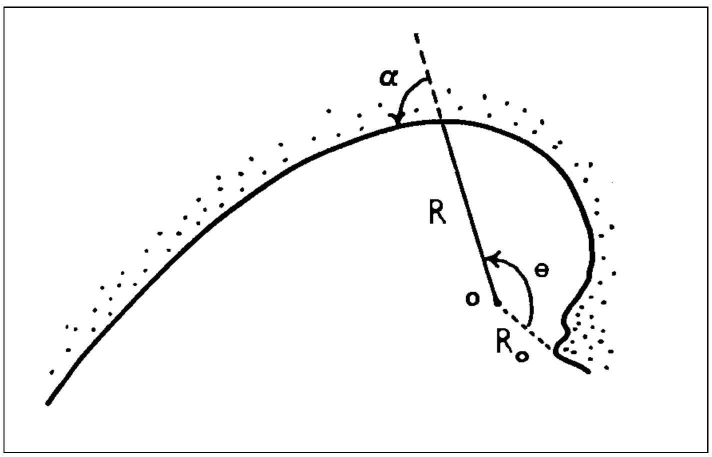

Fig. 1. Definition sketch of the logarithmic spiral shape.

Values of α for headland-bay beaches reported in the literature range from about 45º to 75º. In general, as α becomes smaller, the log spiral becomes wider or more open. There are two singular values or limits for α: if α = 90º, the log spiral becomes a circle, and if α = 0º, the log spiral becomes a straight line.

Sensitivity of the log-spiral shape to small changes in α is shown in Fig. 3. Variations (± 1%) in α-values produce large variations in position of the spiral, because the angle enters the argument of an exponential function. Additionally, the smaller the characteristic angle α, the larger the difference for the same percentage of angle variability. The practical consequence is that, because we are interested in fitting this log-spiral shape to headland-bay beaches – especially in design of shore-protection projects, α has to be accurately defined.

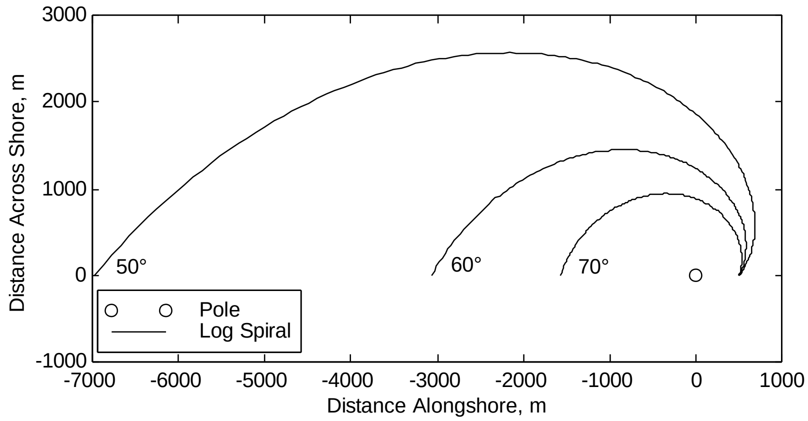

Fig. 2. Variation of logarithmic-spiral shape with α (R_0 = 500 m).

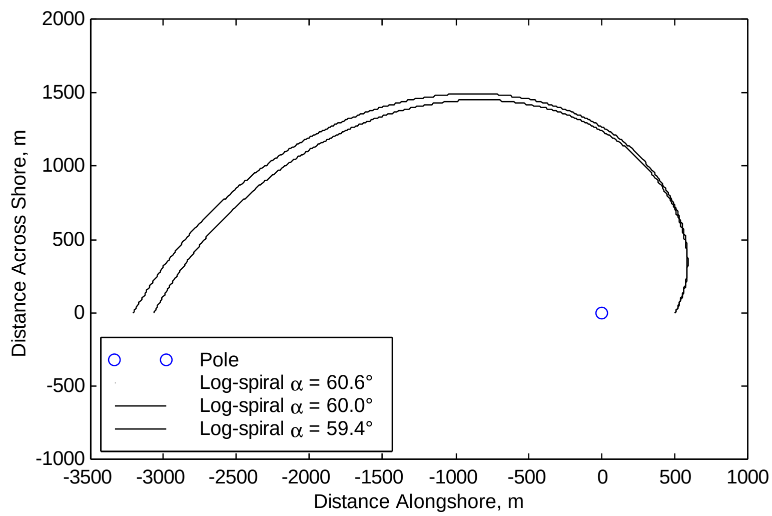

Fig. 3. Sensitivity of log-spiral shape to small variations of the parameter α (R_0 = 500 m).

To analyze the validity of the assumption that headland-bay beaches can be adequately described by a log-spiral function, the pole location, radius at origin of angle measurement (scaling factor), and the spiral characteristic angle α must be identified, giving four unknowns. If the pole position is known, a solution procedure for the best-fit shape, that is, to find the best-fit value of α, follows. Eq. 1 can be rewritten as

$$\ln(R) = \ln(R_0) + q \ \cot a \qquad (2)$$

where R = radius to point P, θ = angle between polar axis and radius vector to point P. This linear relationship is convenient for fitting a log-spiral curve to data. The slope of the straight line is cot α, from which α can be obtained.

A computer program was written that searches for the pole position while minimizing the fitting error. It also determines the characteristic angle α that best fits the shoreline data (measurement) points for a specified bay-shaped beach. The program outputs the following parameters: root-mean square (rms) error of the fit taken in the direction of the radius vector; pole coordinates; angle α; and R_0.

PARABOLIC SHAPE

For the parabolic shape, the focus of the parabola is taken to be the diffraction point. The three coefficients needed to define the shape (e.g., see Silvester and Hsu 1991, 1993) are functions of the predominant wave angle with respect to a control line. The control line is defined similarly to the case of the log-spiral shape as the line that extends from the diffraction point to a reference point. Down drift of the reference point, the shoreline is assumed to be aligned parallel to the incident wave crests. This shape pertains to that of a long straight beach with shape controlled by one headland.

The parabolic shape of a headland bay beach was proposed by Hsu et al. (1987) and is expressed mathematically in polar coordinates by Eq. 3 for the curved section of the beach and by Eq. 4 for the straight down-drift section of the beach,

$$\frac{R}{R_o} = C_0 + C_1 \left(\frac{b}{q} \right) + C_2 \left(\frac{b}{q} \right)^2 \qquad \text{for } \theta \geq \beta \qquad (3)$$

$$\frac{R}{R_0} = \frac{\sin\ b}{\sin\ q} \qquad \text{for } \theta \leq \beta \qquad (4)$$

where R = radius to a point P along the curve at an angle θ; R_o = radius to the control point, at angle β to the predominant wave front direction; β = angle defining the parabolic shape; θ = angle between line from the focus to a point P along the curve and predominant wave front direction; and C_0, C_1, and C_2 = coefficients determined as functions of β. The variable R of the parabolic shape is expressed as a second-order polynomial of β/θ for the curved section of the shape; otherwise, it is a straight line. For $\theta = \beta$, the condition $R = R_0$ must be met, which forces $C_0+C_1+C_2 = 1$ by Eq. 3.

Fig. 4 is a definition sketch for the parabolic shape, and the revised values of the coefficients C_0, C_1, and C_2 are listed in Table 4.2 of Silvester and Hsu (1993). To our knowledge, the *C*-coefficients appear for the first time in the literature in Hsu and Evans (1989). We believe that their values were obtained by fitting of data from 14 bay beaches in Australia and from seven physical-model beaches (Ho 1971). Values of β ranged in prototype beaches from 22.5° to 72.0°, whereas the variation in model beaches was from 30° to 72°. Values of the *C*-coefficients were later revised in Silvester and Hsu (1993) and given in tabular form from β = 20° to 80° at a 2-deg intervals.

As a sensitivity test, we analyzed the response of the parabolic shape to a change of the value of the characteristic angle β and of R_o. The angle ß corresponds to the angle between the control line and the predominant wave crest orientation. R_o is a scaling parameter – the length of the control line.

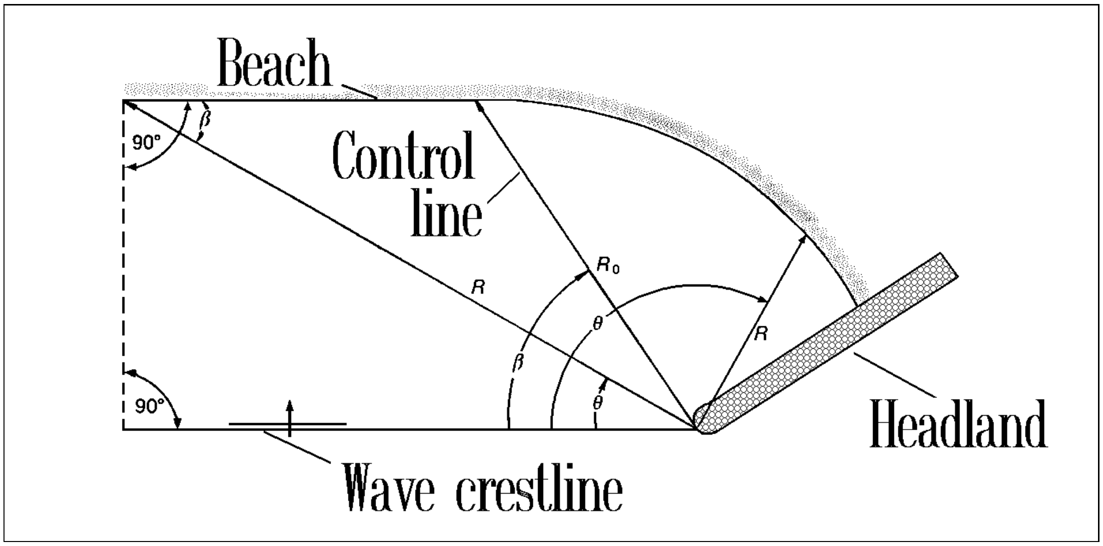

Fig. 4. Definition sketch of the parabolic shape.

Fig. 5 depicts change in parabolic shape for different values of ß for fixed R_0 and a given focus position. Because the parabolic shape is defined only for ? ≥ ß, the alongshore extent of the shape decreases as β increases. In addition, as β increases, the slope of the shape at the point where θ = β departs farther from horizontal. Similarly, Fig. 6 shows the change in parabolic shape for different values of R_o for constant ß, for a given focus position. The scaling effect of the radius R_o is evident. In summary, the angle ß controls the shape of the parabola, and R_o controls its size.

Because the control line intersects the beach at the point where the curved section meets the straight section of the beach, the sensitivity of the parabolic shape to errors in the estimation of the control point was examined. This was done by jointly changing R_o and ß while keeping the distance from the headland to the straight shoreline constant.

Fig. 7 shows the resultant parabolic shapes for slightly different locations of the control point. It is apparent that the parabolic shape is insensitive to ß. This observation means that the control point is not well defined, i.e., uncertainty in selection of the control point and hence the radius R_o and ß has little influence on the final result.

To analyze the validity of the assumption that a parabolic shape can describe headland-bay beaches, the location of the focus of the parabola and the characteristic angle ß must be specified, together with the scaling parameter R_o. Because the parabolic shape is defined in a particular coordinate system, in design applications an extra unknown enters – the orientation of the entire parabolic shape in plan view. Therefore, five unknowns must be solved for in practical applications. For this purpose, a computer program was prepared that solves for the five unknowns by minimizing the composite radial (for the curved section of the beach) and Cartesian (for the straight-line section of the beach) rms error of fit for a given bay-shaped beach.

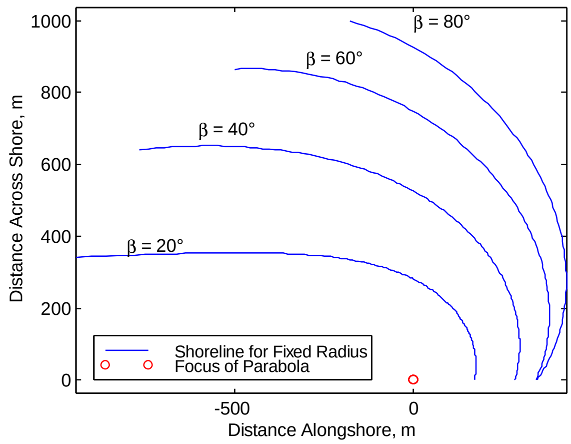

Fig. 5. Parabolic shape for selected values of angle β (R_0 = 1,000 m).

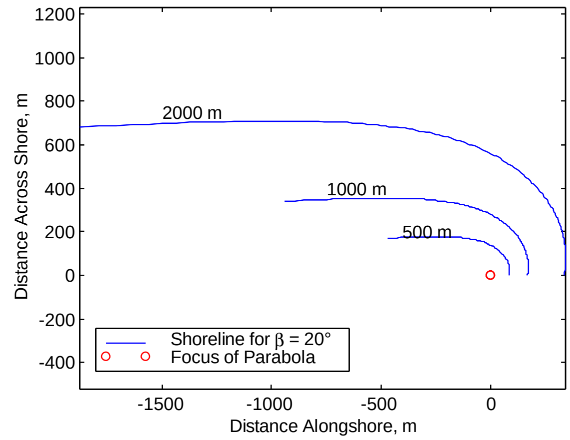

Fig. 6. Parabolic shape for selected values of R_0 (β = 20 °).

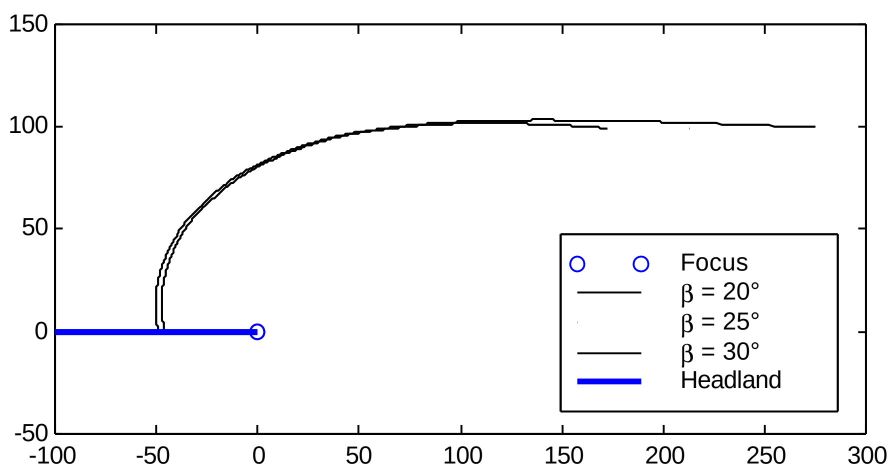

Fig. 7. Sensitivity of parabolic shape to selection of control point location.

If the location of the focus is known, and the wave crests are parallel to the *x*-axis, the values of R and ? can be computed for each data point. For each candidate value of the angle ß, the right-hand side of Eq. 3 can be computed and, therefore, a value of R_o estimated for each point. The selected combinations of focus location, angle of rotation of the local coordinate system to the absolute coordinate system, R_o, and ß are computed such that the radial rms error achieves a minimum.

HYPERBOLIC TANGENT SHAPE

The hyperbolic tangent shape was developed by the authors to simplify the fitting procedure and reduce ambiguity in arriving at an equilibrium shoreline shape as controlled by a single headland. As demonstrated above, it can be difficult to specify the location of the pole or focus, and the characteristic angle (angle between predominant wave crests and the control line) for developing a log-spiral shape or a parabolic shape. In addition, the log-spiral shape does not describe an exposed (straight) beach located far downdrift from the headland, so that another shape must be applied.

Definition

The hyperbolic tangent functional shape is defined in a relative Cartesian coordinate system as

$$y = \pm\, a \tanh^{m}(bx) \tag{5}$$

where y = distance across shore; x = distance alongshore; and a (units of length), b (units of 1/length), and m (dimensionless) are empirically-determined coefficients.

This shape has three useful engineering properties. First, the curve is symmetric with respect to the x-axis. Second, the values $y = \pm\, a$ define two asymptotes; in particular of interest here is the value $y = a$ giving the position of the down-drift shoreline beyond the influence of the headland. Third, the slope dy/dx at $x = 0$ is determined by the parameter m, and the slope is infinite if $m < 1$. This restriction on slope indicates m to be in the range of $m < 1$.

According to these three properties, the relative coordinate system should be established such that the x-axis is parallel to the general trend of the shoreline with the y-axis pointing onshore. Also, the relative origin of coordinates should be placed at a point where the local tangent to the beach is perpendicular to the general trend of the shoreline. These intuitive properties make fitting of the hyperbolic-tangent shape relatively straightforward as compared to the log-spiral and parabolic shapes, making it convenient in design applications.

Properties

Sensitivity testing of the hyperbolic tangent shape was performed to characterize its functional behavior and assign physical significance to its three empirical coefficients. The parameter a controls the magnitude of the asymptote (distance between the relative origin of coordinates and the location of the straight shoreline), and its functioning is self-evident. Fig 8 shows the action of b as a scaling factor controlling the approach to the asymptotic limit. Fig. 9 indicates that m controls the curvature of the shape, which can vary between a square and an S curve. Larger values of m ($m \geq 1$) produce a more rectangular and somewhat unrealistic shape, whereas smaller values produce more rounded, natural shapes.

For small values of x, the defining Eq. 5 is approximated as:

$$y \approx \pm\, a\,(bx)^{m} \tag{6}$$

and the slope dy/dx of the hyperbolic tangent for small values of x is:

$$\frac{dy}{dx} \approx \pm abm\left(bx\right)^{m-1} \tag{7}$$

Therefore, the value of the slope of the hyperbolic tangent at the relative origin of coordinates may be characterized according to the value of the coefficient m:

For $m < 1$, the slope is infinite (gives a symmetry point);
For $m = 1$, the slope is $\pm ab$; and
For $m > 1$, the slope is zero.

Consequently, in practical application, interest lies in values of $m < 1$.

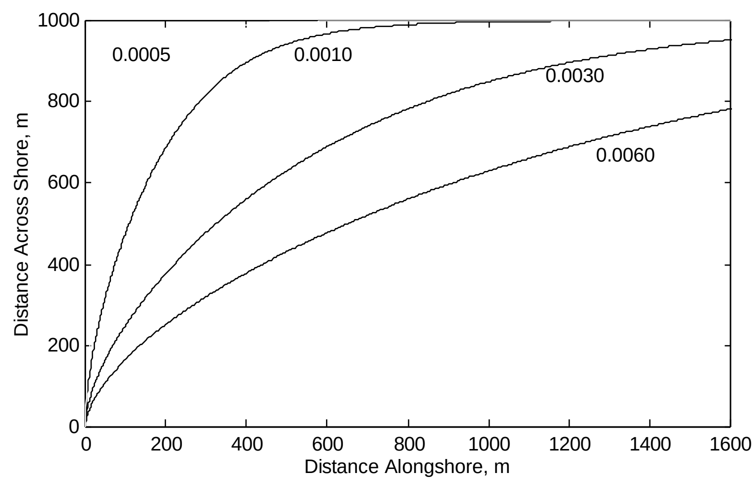

Fig. 8. Dependence of hyperbolic-tangent shape on b (a = 1,000 m; m = 0.6).

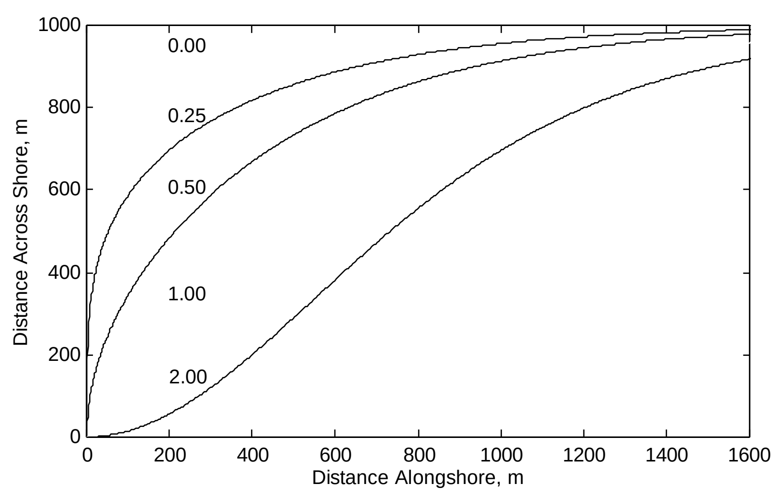

Fig. 9. Dependence of hyperbolic-tangent shape on m (a = 1,000 m; b = 0.0012 m^{-1}).

To fit the hyperbolic tangent shape to a given shoreline, we must solve for six unknowns; the location of the relative origin of coordinates, the coefficients *a*, *b*, and *m*, and the rotation of the relative coordinate system with respect to the absolute coordinate system. Because of the clear physical meaning of the parameters, fitting of this shape can be readily done through trial and error. An optimization procedure that minimizes the rms error with respect to vertical axis values was implemented that solves for the six unknowns.

RESULTS

The three functional headland-bay shapes were fit to the database assembled in this study, as summarized in Table A1. Various authors have noted that fitting of the log-spiral shape is difficult in the down-drift section of the beach, also encountered here. It is a particular concern in attempting to fit to long beaches or to beaches with one headland. However, even in these situations, it was found that a good fit could be achieved for the stretch near the headland.

The parabolic shape provides good fits for beaches with a single headland, because they consist of a curved section (well describes the portion of the beach protected by the headland) and a straight section (well describes the down-drift section). However, this shape is insensitive to values of the determining parameters. Interpolation of the *C*-coefficients over a broad multi-valued plain makes the fitting process time consuming. The possibility of allowing the *C*-coefficients to be free while keeping the second-order polynomial shape has been implemented and will be discussed elsewhere. The goodness of the fit increases significantly in most applications, suggesting re-evaluation of the *C*-values.

The hyperbolic-tangent shape was found to be a relatively stable and easy to fit, especially for one-headland bay beaches. According the fittings shown in Table 1 in the Appendix and the plot of best-fit values in Fig. 10, the following simple relationships are obtained for reconnaissance-level guidance:

$$a\,b \cong 1.2 \tag{8}$$

$$m \cong 0.5 \tag{9}$$

The physical meaning of Eq. 8 is interpreted that the asymptotic location of the down-drift shoreline increases with the distance between the shoreline and the diffracting headland. Eqs. (8) and (9) are equivalent to selecting one family of such hyperbolic tangent functions for describing headland-bay beaches, and these values are convenient for reconnaissance studies prior to detailed analysis.

The mean value of the product *ab* from the database was *ab* = 1.2. Least-squares fitting for the linear function between $\log_{10} a$ and $\log_{10} b$ led to the following relationship shown as the line drawn through the data points in Fig. 10:

$$a^{0.9124}\,b = 0.6060 \tag{10}$$

with a correlation coefficient R^2 of 0.8696.

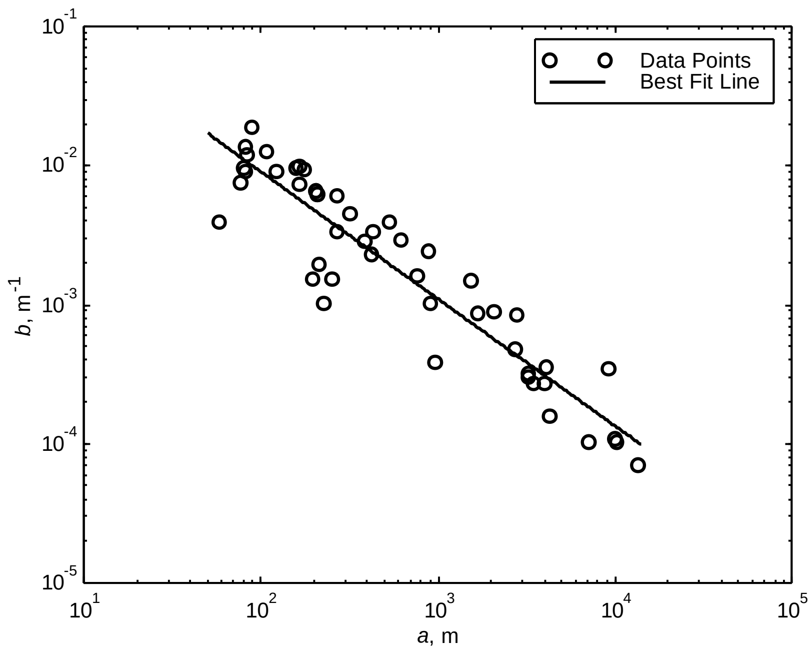

Fig. 10. Plot of *a* versus *b* values for hyperbolic-tangent shape and best-fit line.

EXAMPLES OF FITTING WITH HYPERBOLIC TANGENT

Two examples of fittings of the hyperbolic tangent shape to beaches in the database are shown here. Values of obtained fitting coefficients are listed in Appendix A.

Drakes Bay, California, USA

Drakes Bay is a 16.4-km-long crenulate-shaped beach located north of San Francisco. Its shape is determined by diffraction of Pacific-Ocean waves at a headland called Point Reyes. The area is tectonically active and lies on the San Andreas Fault. Beach sediments consist of discontinuous accumulations of well-to-moderately sorted, fine- to coarse-grained sand interspersed with pebbles and gravel.

Fig. 11 shows the nautical chart with 56 digitized data points and the best-fit curve. The visual fit is very good in the beach section close to the main headland. The curve deviates from the shoreline in the middle section because of irregularities in beach plan form and because of the presence of Drakes Estero (a river mouth).

Rosas Bay, Girona, Spain

Rosas Bay is located on the Northeast coast of Spain. It is a 2.2-km-long crenulate-shaped beach with 1 to 1.5 headlands. Shoreline shape is controlled in great part by jetties. Beach material is fine- to medium-size sand with median grain size in the range of 0.24 to 0.29 mm. Fig. 12 shows an aerial view of the Bay with 29 digitized data points and best-fit hyperbolic tangent. Excellent visual agreement is seen.

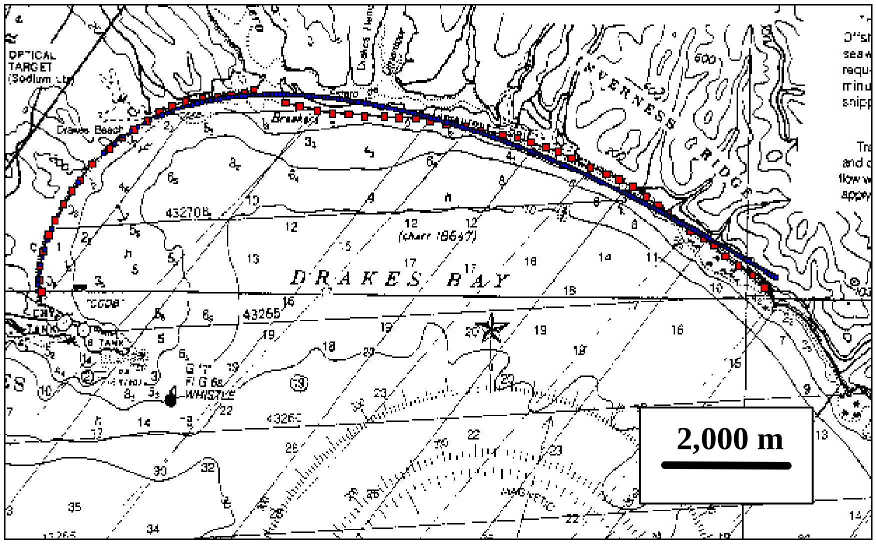

Fig. 11. Hyperbolic-tangent fit to Drakes Bay, California.

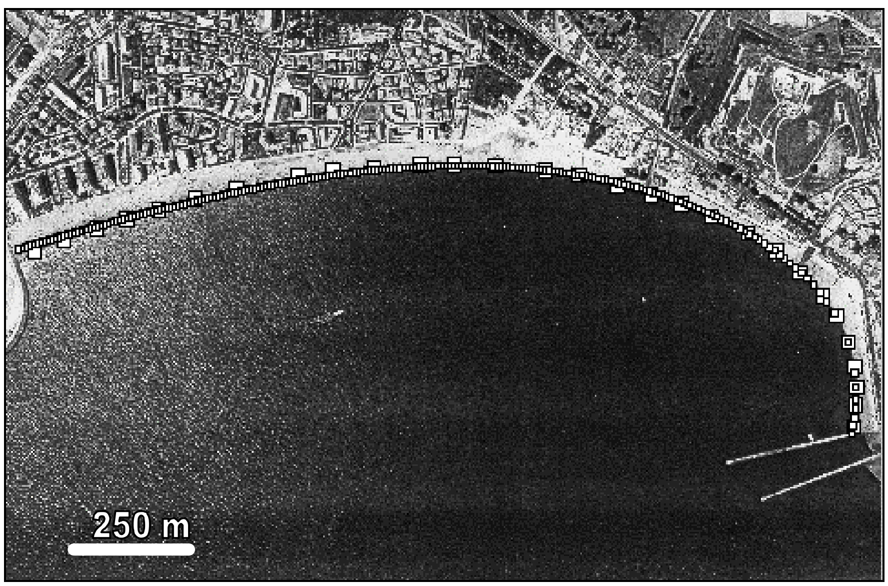

Fig. 12. Hyperbolic-tangent fit to Rosas Bay, Spain.

CONCLUSIONS

A database comprising 23 beaches in Spain and 23 beaches in the North America was developed to examine the equilibrium shapes of headland-bay beaches. Convenient software routines were written to automate the fitting process and make it objective (not discussed in this paper). It is noted that such shapes are applicable to coasts with a consistent predominant direction of net longshore sediment transport.

Based on our study, it is straightforward to fit the log spiral to shoreline-position data, but this equilibrium shape cannot describe beaches with only one headland. In addition, the log spiral is sensitive to the angular parameter defining it. For one-headland beaches, ambiguity exists as to where to terminate the fitting or design. The parabolic shape performs better for beaches with one headland, but it is more difficult to fit than the log spiral and is sensitive to the choice of the control line.

The newly introduced hyperbolic-tangent beach provides good fits for one-headland beaches and is more convenient to apply than the parabolic shape. Guidance was developed for estimating equilibrium shapes with the hyperbolic tangent in reconnaissance studies.

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Appendix A: Summary of Headland-Bay Data Base

Table A1. Summary of fitting of the three headland-bay equilibrium shapes to 46 beaches

	Beach		Log-spiral		Parabolic		Hyperbolic Tangent		
Beach	**No. Head.**	**Length (m)**	**α (º)**	**R_0 (m)**	**β (º)**	**R_0 (m)**	***a* (m)**	***b* (m)**	***m***
Banús (c) Málaga, Spain (c*)	2	348 159	88.26	114	25.63	214	76.7	7.46e-3	0.456
Banús (e) Málaga, Spain (e*)	2	245 157	75.62	68	80.00	152	86.2	1.52e-2	0.544
Benalmádena (d) Málaga, Spain	2	830	74.12	716	75.31	685	533.6	3.91e-3	0.519
S. Antonio Calonge (b) Girona, Spain (b*)	2	325 160	87.86	91	77.78	100	81.3	1.36e-2	0.420
S. Antonio Calonge (c) Girona, Spain (c*)	2	342 167	88.02	105	80.00	127	90.0	1.88e-2	0.617
Cunit (j) Tarragona, Spain (j*)	2	384 173	89.80	116	39.47	135	83.29	1.18e-2	0.410
Cunit (k) Tarragona, Spain (k*)	2	588 298	75.61	276	69.09	216	167.0	7.29e-3	0.364
Drakes Bay, CA, USA	2	16397	38.75	12	33.33	11472	7178.0	1.01e-4	0.444
Kelleys Island, Lake Erie, OH, USA	2	1930	86.12	841	78.84	1273	882.4	2.43e-3	0.813
Point Chehalis, WA, USA	2	1430	52.34	50	39.14	742	385.2	2.86e-3	0.548
Barceloneta (a) Barcelona, Spain	1.5/2	415	82.50	373	75.09	407	320.9	4.50e-3	0.533
Benalmádena (c) Málaga, Spain	1.5/2	645	51.47	59	44.00	458	269.9	5.97e-3	0.439
Campello (a) Alicante, Spain	1.5/2	220	60.38	253	55.24	135	108.8	1.26e-2	0.532
Campello (b) Alicante, Spain	1.5/2	619	82.64	461	67.34	668	614.9	2.88e-3	0.457
Hunters Cove, OR, USA	1.5/2	815	64.98	270	78.58	735	427.4	3.35e-3	0.542
Benicassim, Castellón, Spain	1.5	687	32.47	1002	72.42	353	271.7	3.34e-3	0.355
Bodega Bay, CA, USA	1.5	3200	41.67	227	37.03	2201	1547.9	1.46e-3	0.815
S. Antonio Calonge (d) Girona, Spain	1.5	330	39.47	5	72.96	166	196.8	1.54e-3	0.403
Martha's Vinyard, MA, USA	1.5	4762	68.58	4725	71.54	4210	2731.7	4.80e-4	0.582
Nantucket Is., Coskata, MA, USA	1.5	18999	67.49	12689	80.00	11358	9139.4	3.47e-4	0.655
Nules, (c) Castellón, Spain	1.5	525	59.99	61	49.81	55	164.2	9.70e-3	0.566
Pelee Island (North) Lake Erie, Ontario, Canada	1.5	4231	50.15	2160	43.41	2717	1693.2	8.71e-4	0.665
Sisters Rock, OR, USA	1.5	9376	27.49	6398	62.50	2937	973.5	3.87e-4	0.533
Tituna Spit (S) OR, USA	1.5	10563	23.27	28673	60.00	8001	3231.2	3.04e-4	0.650

Tossa de Mar, Girona, Spain	1.5	336	80.18	488	63.71	155	81.7	8.99e-3	0.531
S. Antonio Calonge (a) Girona, Spain (a*)	1/1.5	947 434	88.13 85.57	1980 495	62.77	127	80.9	9.48e-3	0.334
Halfmoon Bay (18682) CA, USA (5520)	1/1.5	10058 10025	37.68 40.07	5 16474	31.19 20.05	6953 12351	3456.2 3999.4	2.68e-4 2.69e-4	0.466 0.471
Hospitalet del Infante, Tarragona, Spain	1/1.5	2302	32.72	20	36.95	927	422.2	2.25e-3	0.526
Otter Point (*) OR USA	1/1.5	890	39.27	3e-1	46.38	422	254.5	1.54e-3	0.381
Rosas Bay, Girona, Spain	1/1.5	2200	34.46	43	32.41	1608	902.5	1.01e-3	0543
Banús (e*) Málaga, Spain	1	157	76.66	138	79.19	142	204.2	6.62e-3	.195
Barceloneta (b) Barcelona, Spain	1	1023	85.75	2081	64.00	803	212.6	1.93e-3	.280
Benalmádena (c*) Málaga, Spain	1	408	72.62	173	67.66	331	208.2	6.15e-3	.555
Brest Bay, Mich., USA	1	9056	65.41	1006	70.00	5496	4082.5	3.53e-4	.550
Canaveral Bight, FL, USA	1	21911	22.93	24	43.37	15889	13530.3	6.98e-5	0.273
Cherry Harbor, NY, USA	1	5530	78.85	1114	73.74	3332	2819.9	8.47e-4	0.439
Los Cristianos, Tenerife, Spain	1	382 (*) 300	49.86 67.84	531 485	57.19 30.16	229 245	157.7 124.3	9.52e-3 8.95e-3	0.556 0 .322
Hatteras Bight, NC, USA	1	21635	21.95	5	70.67	11104	4256.5	1.56e-4	0.561
La Plaisance Bay, Mich., USA	1	7960	33.25	41	34.05	6141	3238.0	3.14e-4	0.450
SantaMargarita, Girona, Spain	1	1686	87.65	4559	79.99	1160	227.3	1.01e-3	0.475
Napeague Bay, NY, USA	1	3204	60.61	2921	70.91	2698	2097.0	8.88e-4	0.553
Port Clinton, OH, USA	1	24361	54.43	13973	29.50	20150	10345.0	1.01e-4	0.521
Sandy Hook, NJ, USA (*)	1	777	26.96	1077	69.38	245	175.1	9.29e-3	0.497
Shinnecock, NY, USA	1	130	35.10	2e-4	42.04	63	58.2	3.92e-3	0.185
Springhill, MA, USA	1	25444	43.80	1791	21.08	45190	10131.3	1.07e-4	0.674
Tituna Spit (N.) OR, USA	1	2204	70.27	989	20.00	4720	767.5	1.60e-3	0.517

(*) Only the section of the beach close to the headland was selected for fitting. The notation (a), (b), (c), (d), (j), and(k) refers to specific beach cells at the particular location. (18682) and (5520) refer to nautical chart numbers in Halfmoon Bay.